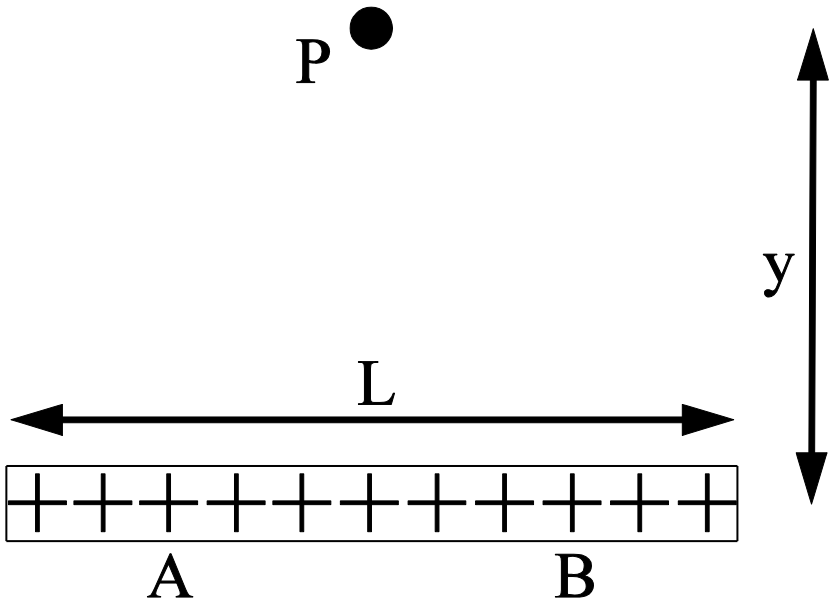
First Name: _____ Last Name:_____

In this tutorial, we will calculate the electric field due to a long straight bar of charge at the point P which is on the bar's perpendicular bisector. While shown as eleven individual positive charges in the figure, in reality the charge is uniformly spread out along the bar. Additionally, the bar is essentially one-dimensional (along the x axis) so the charge distribution can be described by $\lambda = \lambda_0$.

1. Draw vectors representing the electric field at point P due to the charges labeled A and B in the figure. Your vectors should be qualitatively correct.

2. Determine the direction of the net electric field at point P. Explain your answer.



3. To calculate the electric field at P we must evaluate the integral: $\vec{E} = \int \frac{dq\,\hat{r}}{4\,\pi\varepsilon_0 r^2}$. Let's slowly work through this and get the integral set up correctly. How do you express the *dq*?

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4. Your integral certainly needs limits. Two students at your table already have answers. One has $\int_0^L$ while the other has $\int_{\frac{-L}{2}}^{\frac{L}{2}}$. Are both of these answers correct? Will one make the calculation easier than the other? Explain your reasoning.

5. Given that there is an amount of charge *dq* at point A, what is the magnitude of the electric field created by that piece of charge at point P?

6. Determine the y-component of the electric field (since $\hat{y}$ should have been your answer to Question #2) at point P due to the charge at point A. Your work should include a vector diagram illustrating how you determined the component.

7. You should now be ready to write out the integral you need to solve so do it here.

8. Solve that integral for the electric field at point P due to the entire bar. Hint: you will need a trigonometric substitution.

9. Determine the total charge on the bar and rewrite your answer to Question #8 using that.

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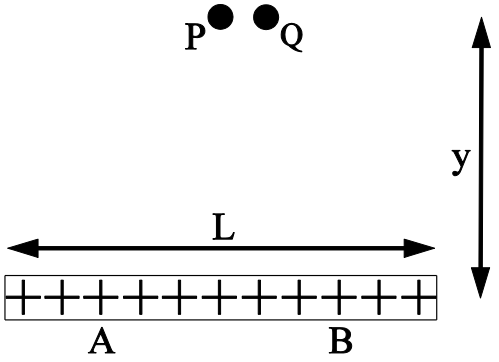
10. Check your answer in Question #9 by verifying that it has the correct units.

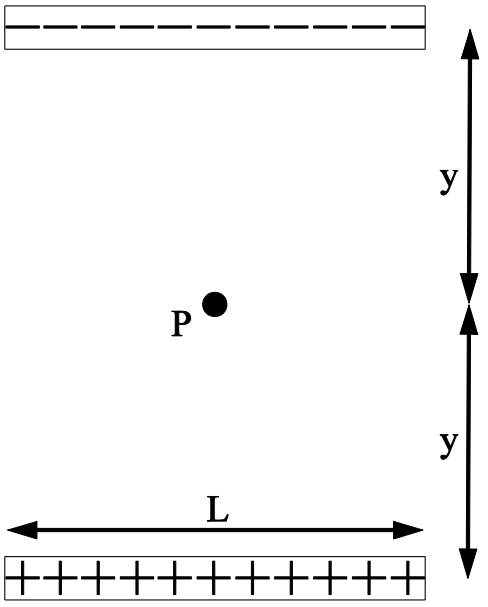
11. What should the electric field at point P be if $y \gg L$? Explain your reasoning. Verify that your answer in Question #9 gives the same result in that limit.

12. A proton is released from rest at point P. Describe and explain its future motion as accurately as possible. You should certainly be referring to its displacement, its velocity and its acceleration and how and why they each change with time.

13. We now want to compare the field at point P with the field at point Q which is not on the symmetry axis. In what ways (if any) will the electric fields be the same? In what ways (if any) will the electric fields be different? Explain your reasoning.



14. Let's modify the problem slightly. There are now two uniformly charged bars: One with $\boldsymbol{\lambda = \lambda_0}$ and one with $\boldsymbol{\lambda = -\lambda_0}$**.** Determine the magnitude and direction of the electric field at point P. Explain your answer.



Comments about this tutorial:

This is intended for calculus-based courses.

During spring 2016 and 2017, we used this on the fifth day of five days covering the first chapter on electrostatics. There's no question that this will always be difficult for the students.

In 2016, my estimate is that basically all groups reached Question #8 but many of them got stuck there for a long time (even with hints from us). A small number of groups did reach Question #14.

In 2017, the vast majority of the students were on Question #8 or #9. Their integration skills are quite poor.

Next time I use this, I am simply going to give them the answers to Questions #8-9 so they can consider some of the more fundamental physics questions which follow.

Changes made 2016:

Question #4: Previously, many many groups were trying to integrate from 0 to L so I rephrased this question and gave them two choices for the limits. Hopefully this will help reduce difficult mathematics later.

Question #13: The figure is new.

Comments about individual questions:

Question #4: Some students like to use $\int_0^L$ since they think that it is beneficial if one of the limits is zero. I've seen this in other situations as well. Many students think that when a limit is zero that the evaluation of the integral at that limit is also zero.

Question #6: They have a lot of trouble with this. We often have them temporarily label an angle in the diagram and then determine the sine or cosine of that angle in terms of the appropriate coordinates.

Questions #10-14: Since very few students have gotten this far, I don't have any specific comments.

Tutorial Source(s):

All questions were written by Drew Milsom.