A Point Charge Near a Grounded Spherical Conductor

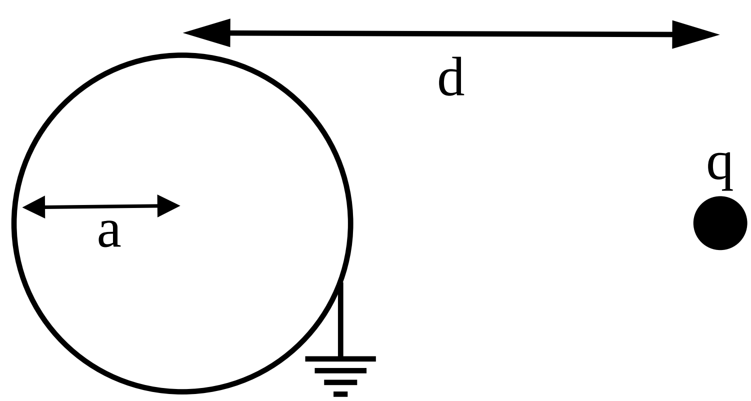
Firstname ______________ Lastname ________________

During this tutorial, you are going to get some practice using an image charge to satisfy boundary conditions.

1: Imagine that you have two point charges near each other as shown. Draw the electric field lines and the equipotential lines (surfaces in 3D) for this situation. Make sure that you indicate which equipotential lines are positive, which are negative and which one is zero.

Your drawing should illustrate that two point charges can create equipotential surfaces which are spherical. Let's use that knowledge here to try and solve the following problem:

A positive point charge q is held in place a distance d from the center of a grounded spherical conductor of radius a as shown. It should be clear that $d \geq a$. We want to determine the electric potential (and electric field, etc.) for all regions outside the sphere. Hopefully, we can accomplish this by using an image charge located inside the conducting sphere. We need that image charge to help satisfy the boundary conditions at the surface of the sphere.



2: List all the boundary conditions on Φ or $\vec{E}$ that apply at the surface of the conductor.

3: Should the image charge (q') be positive or negative? Explain your reasoning.

4: Should the image charge (q') have the same magnitude as q, a larger magnitude or a smaller magnitude? Explain your reasoning.

5: Should the image charge (q') be at the center of the sphere, to the left of center or to the right of center? Explain your reasoning.

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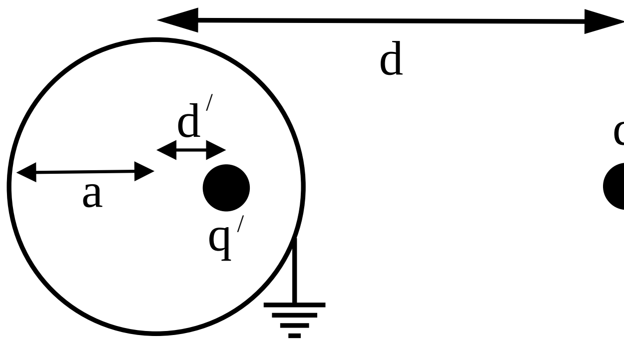
Let's now use this diagram which adds the charge q' a distance d' to the right of the center of the sphere.



6: Let the center of the sphere be your origin and let the line connecting that origin to q be the z-axis. Now express $\Phi(r,\theta)$ for all locations outside the sphere using the sum of the electric potentials due to the real charge and the image charge.

7: We have two unknowns (d' and q') so we need two equations. The equation you wrote in Question #6 must be valid at all θ. Evaluate this at $r = a$ at both $\theta = 0$ and $\theta = \pi$. Then solve those two equations for d' and q'.

8: Evaluate $\lim_{d \to a} d'$ and $\lim_{d \to a} q'$. Do these answers make sense? Explain.

9: Now express $\Phi(r, \theta)$ again using your solutions from Questions #6 and #7.

10: Determine the electric field for all locations outside the sphere.

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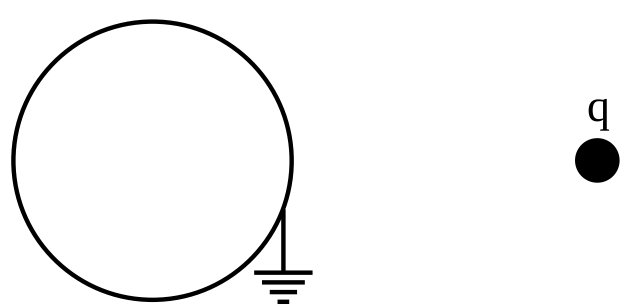
11: Evaluate the θ component of your expression in Question #10 at $r = a$. While it may not appear that this can be simplified, try putting $\frac{d^2}{a^2}$ inside the more complicated of the two expressions which are taken to the $\frac{3}{2}$ power. Does the result make sense? Explain.

12: If you use the radial part of your answer to Question #10 and the radial boundary condition which you wrote in Question #2, you can determine σ_f at $r = a$. Since this is somewhat tedious, here is the answer:

$$\sigma_f(\theta) = \frac{-q(d^2 - a^2)}{4\pi a(a^2 + d^2 - 2ad\cos\theta)^{\frac{3}{2}}}$$

Is σ_f positive anywhere? Does that make sense? Explain.

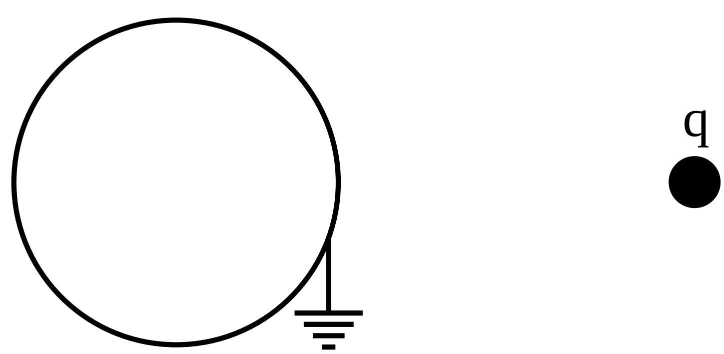
Sketch σ_f as accurately as possible below.



13: Determine the force of attraction between the sphere and the point charge.

14: Evaluate you answer to Question #13 if $d \gg a$. Does your sketch in Question #12 suggest that the conductor looks like a monopole, a dipole, or a quadrupole at large distances? Explain. Is that consistent with the limit you just calculated for the force here? Explain what is going on here.

15: Draw the electric field lines and equipotential lines as accurately as you can.



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Comments about this tutorial:

This was written and used during the fall of 2016. They had 50 minutes and there were six groups of 3-4 students each.

Questions #1-3 - no problems.

Questions #4-5. Some students just assumed it had to be the same size. At some tables, they wanted the charge at the center - which fortunately someone else at their table disagreed with. At one table they clearly wanted to do Question #5 before Question #4 - fine. It does depend upon your logic. If you are thinking about the electric field drawing in Question #1, then Question #4 definitely comes first. If you are thinking about an image charge to the left of center (which is incorrect), you could imagine that the real charge was closer to the right side of the sphere so the image had to be larger in magnitude. It may be useful to tell them to think about both questions at once.

Question #6. The students are still not very good at defining $\vec{r}$, $\vec{r'}$. Many students seem to guess but guess incorrectly. Some had $\sqrt{r^2 - d^2}$. Some did the entire thing in Cartesian coordinates and had some difficulty converting to spherical.

Question #7. I am not sure if anyone got this. They made the algebra far more complicated than it actually is. Not really sure what they were doing wrong. I told a number of groups to continue with the rest of the tutorial.

Question #8. Clearly, the answers were easy here. Their explanations were ok but I didn't see anyone say that this looked like the infinite plane case.

Question #9-10. Two groups got to these but I was working with other students and didn't see how they did.

Two groups were working on Question #10 when time was up.

Changes made 2018:

Question #1: I added the request for them to label sign of the equipotentials. This might make it less likely for them to think that the image charge has the same magnitude as the real charge.

Tutorial source(s):

All questions were written by Drew Milsom. It is based upon a standard image problem which you can find in many textbooks.

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