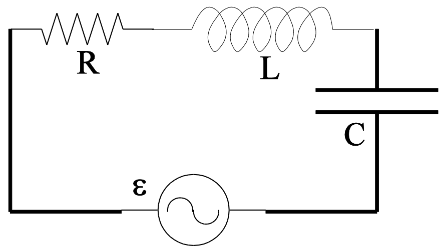
**Driven RLC Circuits**

First Name: ____________________ Last Name:________________________________




You have a series RLC driven by an AC voltage source, $\varepsilon(t) = \varepsilon_{\max}\sin\omega t$. The parameters for the circuit are $R = 96\ \Omega$, $\varepsilon_{\max} = 170$ V, and $\omega = 377$ rad/s. The inductor is a large solenoid with an iron core inside. The iron core can be shifted back and forth to vary the inductance. To begin the problem, use L=0.35 H. The capacitor is also variable so you can turn a knob and vary the capacitance. For now, use $C = 10\ \mu$F. You should also assume that all circuit elements are ideal and remember that $I(t) = I_{\max}\sin(\omega t - \phi)$.

1. Determine the inductive reactance, $X_L$, and the capacitive reactance, $X_C$.

2. Determine the impedance, $Z$.

3. Determine the phase angle, $\phi$.

4. Is this circuit primarily inductive, capacitive, or resistive? Explain your reasoning.

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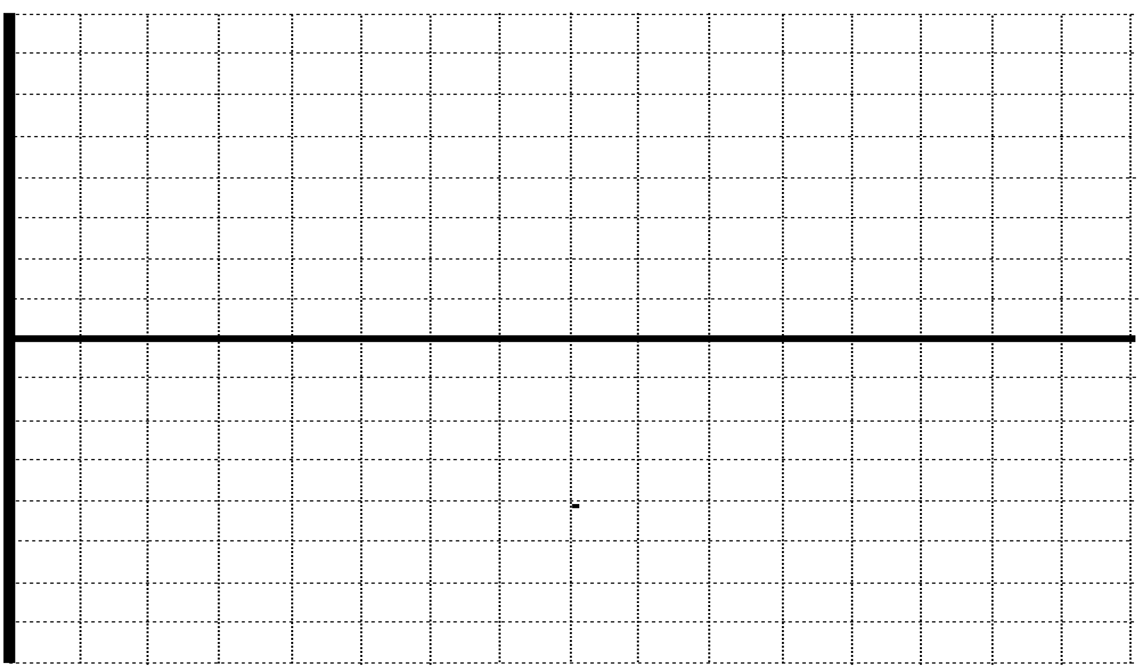
5. To make the circuit dominated even more by the capacitor, would you need to increase the capacitance or decrease the capacitance? Explain your reasoning.

6. Determine the maximum current in the circuit, $I_{\text{max}}$, and write $I(t)$.

7. Graph $I(t)$ and $\varepsilon(t)$ for three complete cycles and label your axes accordingly. Since you are graphing two quantities with different units on the same graph (which is something you may not be accustomed to doing), you can put the scale for $I(t)$ along the left side of the graph and the scale for $\varepsilon(t)$ along the right side of the graph. Is the current leading or lagging the AC source? Explain.



**You will soon be calculating the maximum voltages across each circuit element but before you do any actual calculations let's think about the following two scenarios**.

8. A student uses an oscilloscope to measure the maximum voltage across the capacitor and obtains 225 V. The student thinks they have made a mistake.

   In the student's words, "*Kirchhoff's loop rule states that the three voltage differences must add up to the voltage across the battery. Since the maximum voltage across the battery is only 170 V, there is no way the capacitor could have a voltage drop larger than 170 V*." Is the student's reasoning correct or is there something they don't understand? Explain your reasoning.

9. A student in a different lab group measures the maximum voltage across the resistor and obtains 195 V. This student also thinks they have made a mistake.

   In this student's words, "*I have my own problem here. In my case, I obtained a voltage across the resistor which is larger than the 170 V maximum across the battery. Kirchhoff's loop rule should also imply that this is impossible so I also must have made a measurement error".* Is the student's reasoning correct or is there something they don't understand? Explain your reasoning.

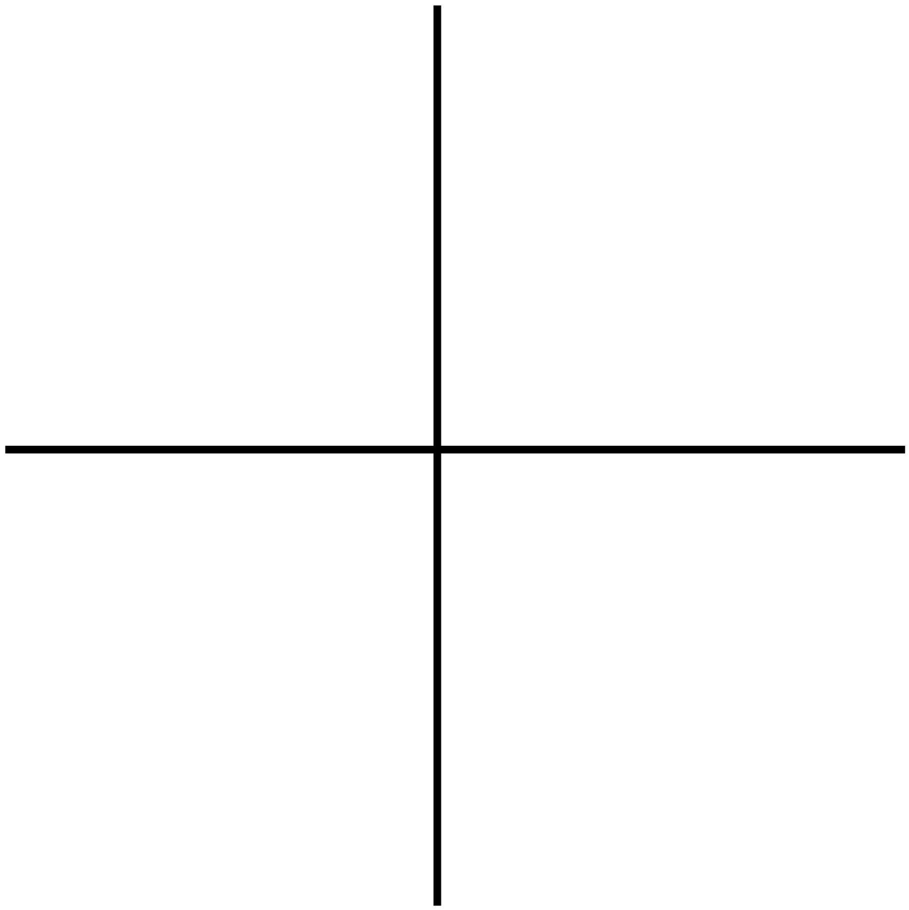
   Hint: Think about how you calculated the impedance in Question #2. What is the minimum impedance associated with the combination of the capacitor and the inductor?

10. Determine the maximum voltage, $\Delta V_{\text{max}}$, across all three circuit elements (R, L, and C). Your answers should all be positive.

11. Determine $\Delta V(t)$ for the resistor, the capacitor and the inductor. To do this, you will need to *derive* these voltage differences using basic principles. Do not use equations from the text or your lab manual. To be consistent with the textbook and our class discussion, if $\Delta V(t) > 0$ then there is a voltage ***drop*** across that particular circuit element.

12. Imagine a time when $\Delta V_C$ is at its maximum positive value so that the voltage drop across the capacitor is as large as it ever gets. Determine the voltage across the battery, the resistor and the inductor at the same time and verify that Kirchhoff's loop rule ($\varepsilon = \Delta V_C + \Delta V_L + \Delta V_R$) is satisfied.

13. Draw a phasor diagram corresponding to the time in Question #12. Don't forget that the ***vertical*** component of your phasor indicates the voltage across that circuit element and that you should be able to use phasor addition to illustrate Kirchhoff's loop rule. You should have a phasor for the voltage across all four circuit elements and one for the current. You should also indicate where the phase angle $\phi$ is.



14. You accidently bump the knob on the variable capacitor and you find that the maximum current in the circuit decreased. Did you increase the capacitance or decrease the capacitance? Explain your reasoning.

15. Is the phase angle in the circuit now closer to $-\frac{\pi}{2}$ rad or is it closer to zero? Explain your reasoning and/or calculate.

**You now set the capacitance back to its original value of $C = 10\ \mu$F.**

16. Remember that the inductor in this circuit is also variable. Determine the inductance you would need to have so that the impedance is at a minimum. In this situation, we refer to the circuit as being in resonance.

**You now adjust the iron core so that the circuit is in resonance.**

17. Determine the rms power delivered to the resistor.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18. Determine the rms power supplied by the AC battery. Is your answer consistent with your answer to Question #17? Explain.

19. We also assumed that all the circuit elements were ideal. However, the inductor has many, many loops of wire. Its resistance is not negligible. Solve Question #17 again if you include the fact that the resistance of the inductor is 60 Ω. Keep in mind that you are calculating the rms power delivered to the actual resistor, not to the additional resistance which is inherent in the inductor.




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**Comments about this Tutorial:**

This is intended for calculus-based courses. The parameters for this RLC circuit actually match one of our in class demonstrations.

During spring 2016 and 2017, they worked on this tutorial during the second of three days covering AC circuits. They had seen how resistors, capacitors, and inductors respond to AC voltages and had then seen the series RLC circuit. We had determined how to calculate the phase angle between the current and the voltage and had defined impedance. At the very end of that single class, we introduced phasors.

The vast majority of the groups were between Questions #9 and #12 after 50 minutes. Approximately half of the groups were on Questions #11 or #12. Several slow groups were on Question #7 and the fastest group was on Question #14.

This was also used in a class of physics majors during the spring of 2018. All seven groups got to at least Question #12.

**Comments about individual questions:**

Questions #1-3: Clearly very straightforward.

Question #4: While the definition of primarily inductive or primarily capacitive is fairly obvious, many students asked about what primarily resistive was. You could say that $R > |X_L - X_C|$ or similarly you could say that the phase angle is close to zero. It doesn't matter here but I am just pointing out that this came up in conversations with them.

Question #5: This is certainly very confusing to them. Most answer the question correctly because they see that capacitance is in the denominator but they certainly can't explain what that really means. If you say to them "Wouldn't you expect that if the capacitance is larger than the capacitor should be more important?" They usually say yes to this. You can also mention that this is similar to the fact that in series the smaller of two capacitors would store more energy. If you engage the students here, you can initiate some interesting discussions.

Question #7: It is clear that they need lots of practice graphing.

Questions #8-9: They did much better on these two in 2017 than in 2016.
It is not just because of the changes referenced below. I am not entirely sure why.

**Changes made after 2016 class:**

Question #7: They were extremely confused about graphing things with different units particularly since one has a much larger magnitude than the other. I added the comment




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about labeling the left and right sides. It is possible that they have never seen graphs of two different quantities like this.

Question #9: While a reasonable number of students understood Question #8, very few understood Question #9. I have added the hint.

Question #11: At least half the students were using equations from our lab manual so I made it clear that they must derive this.

**Tutorial Source(s):**

All questions were written by Drew Milsom.


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