Disoriented Chiral Condensate

Ina Sarcevic and Sean Gavin

The primary aim of present and future heavy ion experiments is to establish properties of the QCD phase transition. If the chiral transition is nearly second order, Rajagopal and Wilczek propose that the nonequilibrium dynamics in ion–ion collisions at RHIC can generate transient domains in which a macroscopic pion field develops [1, 2]. These domains are called disoriented chiral condensates (DCC). Bjorken, Kowalski, Taylor and others pointed out that DCC can lead to fluctuations in the charged and neutral pion spectra [3]. In nuclear collisions, the ability of experimenters to identify DCC amidst the background due to conventionally produced particles critically depends on the size and energy content of the domains.

Equilibrium high temperature QCD manifests a chiral symmetry if the light up and down quarks are taken to be massless. However, a phase transition occurs at a critical temperature $T_c \sim 150$ MeV in which chiral symmetry is broken by the formation of a scalar $\langle \overline{q} q \rangle$ condensate. In the real world chiral symmetry is explicitly broken at all temperatures by the few–MeV current quark masses. In that case, the nature of the transition has yet to be established.

Rajagopal and Wilczek pointed out that the chiral condensate can become temporarily *disoriented* in the nonequilibrium conditions encountered in heavy ion collisions [1]. As the temperature drops below T_c, the symmetry can begin to break by developing domains in which the condensate is misaligned in the isospin space. The misaligned condensate has the same quark content and quantum numbers as do pions, and essentially constitutes a macroscopic pion field. In a heavy ion system the evolving domains can radiate pions preferentially according to their isospin content. The excitation of a pion field over a substantial fraction of the collision volume can then result in novel fluctuations in the number of neutral and charged pions. If the pion field produced in nuclear collision were uniform over the collision volume, then the fraction of neutral pions f radiated from a series of such collisions would satisfy the probability distribution

$$P(f) \sim 1/\sqrt{f}. \tag{1}$$

On the other hand, if the pion field is not uniform, the nuclear collision instead produces many uncorrelated small domains, the spectrum of fluctuations would be gaussian, as expected, *e.g.*, if the pions were produced independently.

To develop a concrete picture of how the chiral condensate evolves in a nuclear collision, most authors have used the linear sigma model [1]. In this model, the pion field is coupled to a scalar σ field that characterizes the scalar condensate. These fields interact through a potential

$$V = \lambda(\vec{\pi}^2 + \sigma^2 - v^2)^2/4 - H\sigma. \tag{2}$$

Chiral symmetry transformations correspond to O(4) rotations of the vector field $\vec{\Phi} = (\sigma, \vec{\pi})$.

For $H = 0$ this symmetry is spontaneously broken at zero temperature, with $|\langle\vec{\Phi}\rangle| = v$. As the temperature is increased, this symmetry is restored in a second order phase transition in the same universality class as the O(4) Heisenberg model and QCD. The mean–field estimate for the critical temperature of this transition is $T_c \sim \sqrt{2}v$. Aspects of the equilibrium statistical mechanics of this model are treated in refs. [4]. For $H \neq 0$, the symmetry is explicitly broken at all temperatures and there is no phase transition in the strict thermodynamic sense; this may well be the case in QCD also. However, for a sufficiently small H, the thermal–equilibrium value of the condensate is reduced from $\approx v$ to a value $\langle\sigma\rangle \sim (H/\lambda)^{1/3}$ at $T = \sqrt{2}v$ (again in the mean–field approximation).

In the 'quench' scenario proposed by Rajagopal and Wilczek, one assumes that the system initially has $\Phi = 0$, as appropriate at high temperature, and follows the development of the system using $T = 0$ equations of motion derived from V. The initial state of the system is clearly unstable. The system "rolls down" from the unstable local maximum of $V(\Phi)$ towards the nearly stable values with $|\Phi| = v$ (the symmetry breaking term $-H\sigma$ is relatively small). Field configurations with $\vec{\pi} \neq 0$ develop during the roll–down period. The field will eventually settle into stable oscillations about the unique vacuum $(f_\pi, \vec{0})$ for $H \neq 0$, but oscillations continue until interactions eventually damp the motion.

Gavin, Gocksch and Pisarski studied the size and f distributions of domains using numerical simulations of the linear sigma model equations of motion. Linearized equations of motion imply that the average domain size is roughly [5]

$$R_D \sim 2\tau_R \propto m_\sigma^{-1}. \tag{3}$$

For $m_\sigma = 600$ MeV, the pion field in simulations exhibit structure that grow to a size of order $2-4$ fm the quench scenario [5, 6, 7, 9]. Observe that the notion of domains of a fixed 'size' is not rigorously defined in this system – the pion field is inhomogeneous and the scale of inhomogeneities are on average $\sim 2-4$ fm.

In ref. [5] we studied the distribution of the fraction of neutral pions with fields $\vec{\pi}$ obtained by simulating the fully nonlinear classical field equations in 3+1 dimensions. Our simulations were performed on a lattice of a volume relevant to a heavy ion collision, $40\times10\times10$ fm^3. To study the dependence of the results on the sigma mass, calculations were performed for both the normal free–space mass, $m_\sigma = 600$ MeV, and for a very light mass, 2 MeV. In [5], as in [1], a 1 fm lattice spacing was used to simulate the fact that short–wavelength fluctuations are stable.

The lesson drawn from the quench scenario is that large nucleus–sized domains can occur if the sigma mass is small. It is quite possible that the effective m_σ is reduced from the 600 MeV free space value in the high energy density heavy ion environment [6, 7]. In fact, at a truly second order phase transition one expects m_σ — the inverse correlation length — to vanish! In that case, the parameters of the effective potential for our model cannot be *naively* taken from the zero temperature Gell–Mann—Levy model as we have done.

To explore the role of the medium in domain formation, Gavin and Müller studied the dynamical evolution of the condensate in the presence of a nonequilibrium bath of quasiparticles [8]. The three-dimensional expansion of the heat bath changed the effective potential rapidly enough to create a quench-like condition. The size scale was for domains was found to be somewhat larger, perhaps $\sim 3-6$ fm, compared to fixed-geometry quench simulations [5] and one-dimensional Bjorken-scaling expansion [9]. This result has been verified by simulations by Randrup [10]. [This use of the term "annealing" in ref. [8] is misleading. Annealing simulations in [9] simulate true annealing, where both the order parameter

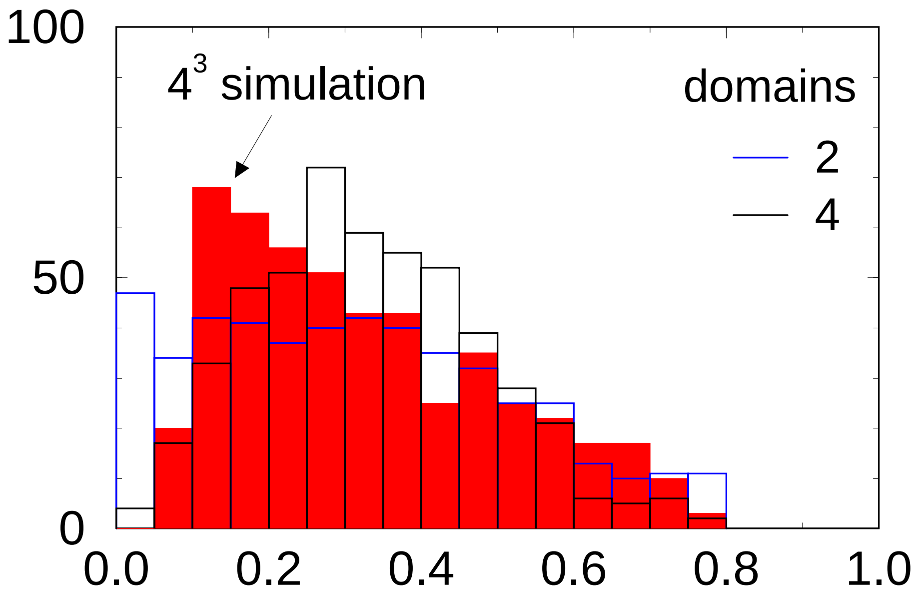

Figure 1: The probability distribution for the fraction of neutral pions f calculated in the linear sigma model.

and quasiparticles are essentially in thermal equilibrium. In ref. [8], the order parameter is assumed to be out of equilibrium while the quasiparticles are in equilibrium.]

The properties of strongly-interacting matter can change drastically as the temperature is raised above a value $T_c \approx 150$ MeV, perhaps undergoing a phase transition. The light up and down quarks have masses of about 7.5 MeV, much smaller than this temperature. On the other hand, the strange quark has a mass of about $m_s \sim 150$ MeV, coincidentally very close to T_c. In the chiral limit with massless light quarks, the phase transition would be second order if the strange quark mass were much larger than T_c, or first order if the mass were much smaller. If this regime describes real world QCD, then disoriented chiral condensates are likely to be observed at RHIC.

How well does the O(4) linear sigma model describe high temperature QCD? Gavin, with Gocksch and Pisarski, pointed out that, if we could vary m_s as a parameter, we would eventually reach a critical point, analogous to that in the liquid-gas phase transition. Specifically, they addressed the nature of the chiral transition in [14], studying the dependence of QCD on m_s in the SU(3) linear sigma model. Increasing the quark masses but keeping the strange-to-light-quark fixed at the natural value ~ 20 decreases the strength of this first order transition until a second order chiral critical point is reached. We find that a scalar $\sigma_{\eta'}$ meson — the parity partner of the η' — becomes massless at that point. Together with recent lattice simulations, these results provide the first indication that QCD in the real world can be near a Z(2) critical point in the universality class of the liquid-gas system. Whether or not the real world is near the chiral critical point remains an issue of current debate in the lattice QCD community. It is not clear from lattice results which side of the critical line we are on.

We start at zero temperature by fitting the scalar and pseudoscalar mass spectrum in QCD to that found in a linear sigma model. For three quark flavors we introduce the field Φ

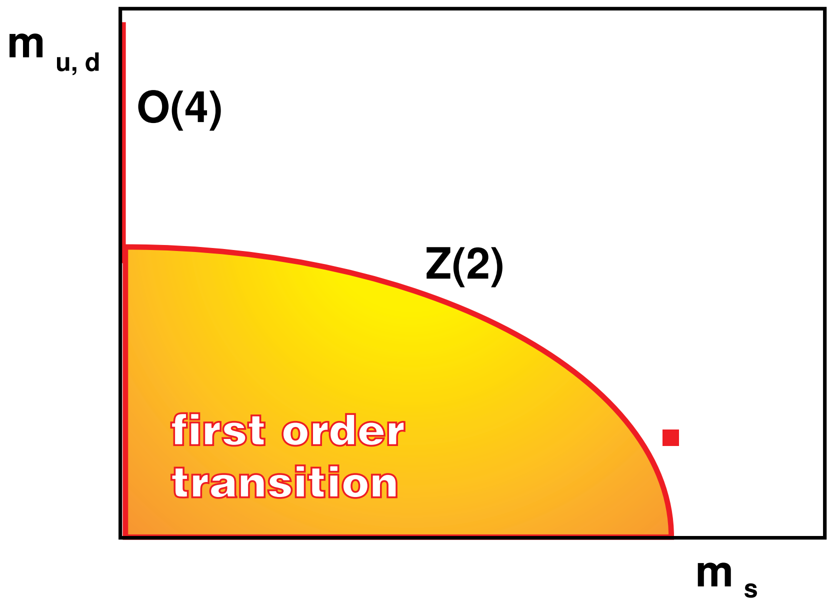

Figure 2: Columbia plot [16] showing the order of the QCD phase transition as a function of the light and strange quark masses. The shaded area denotes values for which the phase transition is first order. The phase transition is second order along the red lines. The red square indicates the likely mass values in the real world.

($\sim \overline{q}_{left}\, q_{right}$), as a complex valued, three by three matrix, $\Phi = \sum_{a=0}^{8}(\sigma_a + i\,\pi_a)t^a$; $t_{1\ldots 8}$ are the generators of the $SU(3)$ algebra in the fundamental representation, and t_0 is proportional to the unit matrix. The effective lagrangian for the Φ field is taken to be [15]

$$\mathcal{L} \;=\; tr\,|\partial_\mu \Phi|^2 \;-\; tr\left(H(\Phi + \Phi^\dagger)\right) \;+\; \mu^2\, tr\left(\Phi^\dagger \Phi\right) \tag{4}$$

$$-\;\sqrt{6}\,c\left(det(\Phi) \;+\; det(\Phi^\dagger)\right) \;+\; (g_1 - g_2)\left(tr\Phi^\dagger\Phi\right)^2 \;+\; 3\,g_2\, tr\left(\Phi^\dagger\Phi\right)^2$$

The parameters of the linear sigma model are the background field H, a mass parameter μ^2, an "instanton" coupling constant c, and two quartic couplings, g_1 and g_2. Neglecting effects from $m_u \neq m_d$, for the background field H we take $H = h_0\, t_0 - \sqrt{2}\, h_8\, t_8$. The current quark masses are then related to the background field as $m_u = m_d \sim h_0 - h_8$ and $m_s \sim h_0 + 2\,h_8$. Our $T = 0$ results agree with those of Chan and Haymaker [15].

We then explore the nature of the phase diagram as a function of the quark masses $m_u = m_d$ versus m_s [16]. We find that along the entire line of chiral critical points, only the $\sigma_{\eta'}$ is massless, but that the singlet/octet ratio in the $\sigma_{\eta'}$ changes. At the $SU(3)$ symmetric point, $m_u = m_d = m_s$, the $\sigma_{\eta'}$ is an $SU(3)$ singlet. Decreasing m_s to the critical point where $m_s = 0$ and $m_u = m_d \neq 0$, the $\sigma_{\eta'}$ becomes entirely strange, $\sigma_{\eta'} \sim \overline{s}s$.

The opposite limit of $SU(2)$ chiral symmetry, $m_u = m_d = 0$, is more familiar. Assume, as in ref. [16], that the chiral phase transition with two massless flavors is of second order, with the universality class that of an $O(4)$ critical point. There is then a special value of $m_{strange} = m^{crit}_{strange}$, with a line of $O(4)$ critical points for $m_s > m_s^{\rm crit}$, and a line of first order transitions when $m_s < m_s^{\rm crit}$.

At the time [14] was written, parameters for the linear sigma model were fit by identifying the mass of the $\sigma_{\eta'}$ with that of the $a_0(980)$. However, since then the Particle Data Group has recently added a $\sigma(600)$ to its list of particles. This meson may possibly correspond to Rajagopal and Wilczek's SU(2) symmetric sigma meson, *i.e.* the chiral partner of the pion. Certainly, feeding that identification into our analysis will change the results, and can have important implications regarding whether the real world is closer to O(4) or Z(2) behavior.

Suppose now that we make a DCC — how can we tell? Let us focus on the O(4) case. Disoriented chiral condensate production results in a fluctuation of the number of charged and neutral pions at low transverse momentum, perhaps $< 100 - 200$ MeV. The signature of this phenomenon is a probability distribution of the fraction of of neutral pions that is broader than a gaussian. The task of discovering DCC therefore amounts to determining charged and neutral pion distributions at low p_T either in individual events or in event classes (superevents).

Gavin obtained the estimate of the magnitude of the DCC signal in heavy ion collisions by supposing that each collision is dominated by a single large domain [11]. In this case, the number of pions is determined by the amount of energy available for pion production per unit volume, which is essentially the potential difference $\Delta V \sim 60$ MeV fm^{-3} between $\Phi = 0$ and $\Phi = (f_\pi, \vec{0})$. If all of the energy in a domain of size $\xi \sim 3 - 4$ fm goes into pion production, one expects $N_{\rm DCC} \sim \Delta V \xi^3/m_\pi \sim 30 - 100$ pions. In comparison, conventional production mechanisms in a central Au+Au collision at RHIC can produce ~ 350 neutral pions per unit rapidity; this constitutes a background to the DCC signal.

The source of pions from this domain is coherent and, therefore, concentrated at low transverse momenta. Pions are radiated from the coherent state [12] describing such a domain give rise to a distribution:

$$E\left(\frac{dN}{d^3p}\right)_{\rm DCC} \propto {\rm e}^{-r^2/R^2}, \tag{5}$$

where R is the size scale of the domain. Coherent production from the domain adds to incoherent production from the underlying event (the "heat bath"), so that the total distribution is

$$E\left(\frac{dN}{d^3p}\right)_{\rm tot} = E\left(\frac{dN}{d^3p}\right)_{\rm background} + E\left(\frac{dN}{d^3p}\right)_{\rm DCC}. \tag{6}$$

For a background distribution taken from the HIJING event generator [13], the calculated distribution is shown in Fig. 10.

Gavin has distributed a monte carlo event generator that produces charged an neutral pions from a single large DCC to aid RHIC experimenters in testing their event-by-event sensitivity to low momentum pions. This generator has been used by RHIC PHENIX, STAR and PHOBOS, CERN WA98 and AGS E864 and E866, to name a few.

New excitement surrounds the possibility of looking for event structure, whether by wavelet analysis or by lego plots. Generally, a collision can form many domains with a range of sizes, rather than a single dominant structure. The distribution that gives rise to EdN/d^3p is then broader and without the bump at low p_T. This is not a "bug" as first feared, but a "feature" — a feature that one can search for. Specifically, the spatial structure of DCC events can translate into momentum space structure that experiments can probe for.

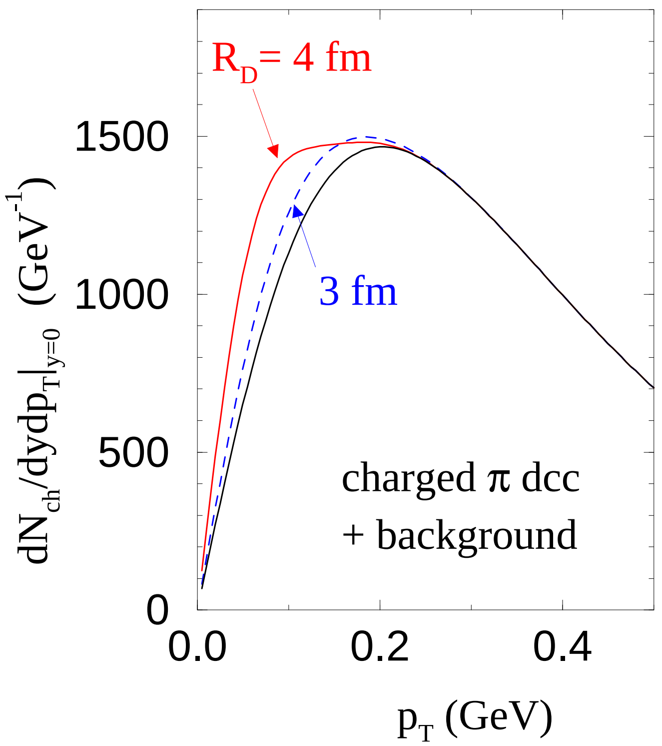

Figure 3: The distribution of charged pions from a single large DCC of sizes 3 fm (dashed curve) and 4 fm (solid curve) in Au+Au at RHIC. The background (heavy solid curve) is estimated using HIJING.

Future work:

To help experimenters develop the more sophisticated analysis tools needed to search for this structure, Rajagopal and Gavin are developing an event generator that generates the DCC signal pions using occupation numbers generated from classical field simulations. These are then added to a background from HIJING. This prescription shares much of the ad hoc nature of my earlier generator. However, there remain many obstacles to building a theoretically well founded event generator for RHIC experiments, and the time for beam at RHIC is coming closer.

How would an approximate Z(2) transition differ from an approximate O(4) transition? What are the phenomenological implications for heavy ion collisions? If the real world is near the chiral critical point, heavy ion collisions can result in nonequilibrium behavior of the $\sigma_{\eta'}$ field. Because this field has $s\bar{s}$ as well as light-quark content, its fluctuations can result in both kaon and pion fluctuations. This behavior is different from the 'familiar' disoriented chiral condensates, which correspond to a world in which QCD has O(4) chiral phase transition. The formation of O(4) DCC affects only pion production. Moreover, nonequilbrium behavior near a Z(2) critical point will shift the masses of the η and the η', leading to enhanced photon production [17]. We intend to investigate the phenomena in the context of the linear sigma model.

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Wavelet Analysis of the DCC in Heavy-Ion Collisions

(*Ina Sarcevic, Zheng Huang, Xin-Nian Wang and R.L. Thews)*

Detection of DCC [1], would provide valuable information on the vacuum structure of the strong interaction and the nature of the chiral phase transition. Preliminary theoretical investigations on non-equilibrium dynamics using the classical linear σ-model have found some evidence for the growth of long wavelength pion modes, which may indeed lead to domain structure or cluster formation [2, 3]. If there are many uncorrelated small domains, the integrated probability distribution of the neutral pion fraction f emitted from a disoriented region, predicted to be $P(f) = 1/2\sqrt{f}$, [1] would become Gaussian. This makes the experimental search for the DCC signal a rather difficult task. In order to disentangle the DCC domain structure in high energy heavy-ion collisions, we propose a new method which emphasizes not only the behavior of the probability distribution in the full phase space region but also its fluctuation in rapidity η or azimuthal angle ϕ. In other words, one needs to study the "local" properties of the distribution in phase space if the DCC clusters are "localized" objects in phase space.

A novel method is a multi-resolution analysis performed by a discrete wavelet transformation (DWT) [4] which has been found effective in systematically detecting structures of various scales in turbulence, and multiparticle production.[5, 6] For any one-dimensional sample of a point distribution $f(x)$, such as the rapidity distribution of the neutral pion fraction in the interval $[0, 1]$ with resolution Δx, can be represented as a histogram of 2^J bins where $J = \mathrm{mod}(|\ln\Delta x|/\ln 2) + 1$

$$f(x) \equiv f^{(J)}(x) = \sum_{k=0}^{2^J-1} f_{Jk}\phi_{Jk}(x), \tag{7}$$

where $\phi_{Jk}(x)$ are the piecewise constant functions called the mother functions. The histogram function at the next finer resolution is the average of adjacent bins at the finest scale. Evidently some detail is lost compared to (7). The difference between (7) and $f^{(J-1)}$ can be fully expressed in terms of the difference functions $\psi_{J-1,k}(x)$ which are stairwell functions called father functions. The difference amplitudes or one defines the DWT coefficients

$$\tilde{f}_{jk} = 2^j \int f(x)\psi_{jk}(x)dx, \tag{8}$$

where the basis functions $\psi_{jk}(x)$ have a unique property: they are related to each other by scaling and translation: $\psi_{jk}(x) = \psi(2^j x - k)$. The index j denotes the resolution scale and k the position at each scale. The idea of multiresolution analysis is to find representations of the sample function $f(x)$ at various scales. Unlike the Fourier transformation which requires the information in whole physical space, the wavelet SSD requires only the local information in space. Roughly speaking, the multiresolution analysis is a journey down (or up) a hierarchy of scales and to view the "world" in different "eyes" with magnifying or reducing glasses.

The central idea of using the wavelet analysis is to attempt an unbiased separation of the DCC physics from the various "backgrounds" which cannot be otherwise easily eliminated. The usual correlation method is not so effective here because the different scale correlations are mixed together in the same phase space. The discrete wavelet transformation (DWT), on the other hand, has an extra dimension – the scale which can be used to separate these correlations. The main source of the "backgrounds" is the fluctuations at small correlation scale due to pion radiation from the uncorrelated regions in space-time, which can coexist with a DCC domain. The background can also come from the partial thermalization of the system leading to thermal correlations.

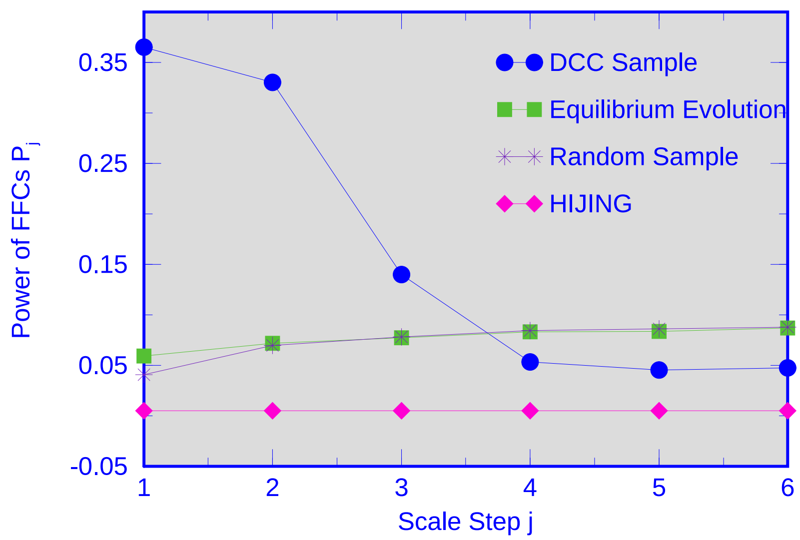

Figure 4: The power spectra of the wavelet coefficients for different dynamical scenarios.

We have calculated the wavelet power spectrum for rapidity distribution of the neutral pion fraction simulated using the classical linear σ model [3]. One defines the power spectrum with respect to the wavelet basis at each j as

$$P_j = \frac{1}{2^j} \sum_{k=0}^{2^j-1} |\tilde{f}_{jk}|^2 \ . \tag{9}$$

P_j is then the power of fluctuations on length scale $L/2^j$. It is also the mean square of the probability distributions of the wavelet coefficients defined in (8). The result is shown in Fig. 11. For the random samples and the equilibrium evolution of the classical field where there is no DCC production mechanism, the wavelet power is flat indicating there is no sensitivity on the change of the scale. For the DCC samples, the power starts to build up as j goes down below $j_d \sim 4$ and rises rather quickly, departing distinctively from the equilibrium and the random cases. This suggests the existence of the DCC clustering with a typical size of $\Delta\eta_d = 2\eta_{\max}/2^{j_d} \simeq 0.8$ units in rapidity. Also plotted is the power spectrum from HIJING Monte Carlo data which also features a flat distribution.

Our wavelet analysis has attracted a lot of attention and has been already used by WA98 Collaboration for studying DCC formation in Pb+Pb collisions at CERN [7]. Wavelet analysis will also be used by the STAR Collaboration at RHIC, whose preliminary studies are shown in Fig. 12 below.

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Event Structure of DCC

(*Sean Gavin*)

Gavin with R. Bellwied and T. Humanic, collaborators at RHIC experiment STAR, are now exploring the effect of collective flow and final state scattering on DCC signals [1]. We can see the importance of flow by considering the following illustration. At midrapidity in a central symmetric heavy ion collision, the matter tends to flow radially outward from the origin. Suppose that as the system cools below T_c, a DCC forms with a field configuration such that it will radiate only charged pions for $x > 0$ and only neutral pions for $x < 0$. At each point the DCC will have the same average radial velocity as the underlying matter (in the matter's local rest frame, the pion field has no preferred direction). Therefore, the charged pions exhibit a net flow towards $x > 0$. An appropriate experiment would detect directed flow towards $+x$. Such flow is entirely unexpected because the collision system is symmetric, *i.e.* the impact parameter is zero. More generally, radiation from DCC can produce pions that reflect the velocity field of the underlying fluid, thereby resulting in an apparent azimuthal anisotropy.

To study the effect of flow in a realistic setting in which pion scattering can occur, Bellweid, Gavin and Humanic combine Gavin's monte carlo DCC generator [2] with Humanic's hadronic cascade code [3]. The cascade assumes an initial distribution of mesons with a Bjorken geometry, but then allows them to scatter with cross sections taken from phenomenology. Scattering tends to absorb or thermalize pions from DCC domains produced in the interior of the collision volume. The following figure shows the effect of scattering on DCC centered at the origin as seen in a wavelet analysis. The STAR acceptance is assumed. For a DCC in the center of a collision volume, none of the flow effects are present. Nevertheless, the wavelet analysis still picks out a signal.

In contrast, we expect that the anisotropy resulting from a moving, off-center domain to actually be *enhanced* by the scattering of DCC pions. Pions with momenta oriented towards the center of the collision region have a greater chance of scattering than outbound pions. Inbound pions are likely to rescatter, while outbound DCC pions have a good chance of escaping without scattering.

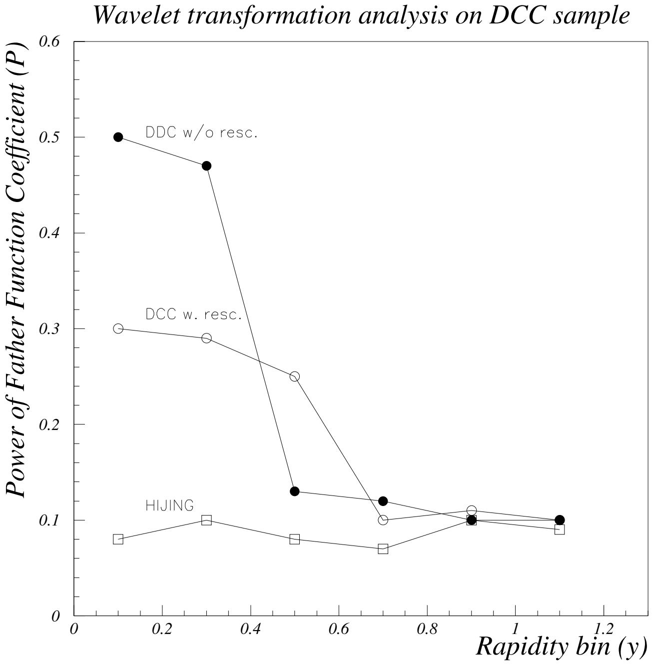

Figure 5: Wavelet analysis from Bellwied, Gavin and Humanic for a DCC in the center of a simulated Au+Au collision at RHIC with cascading included.

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HBT from DCC

(Sean Gavin and and Ina Sarcevic)

We plan to study the question of how DCC influences the Bose correlations of pairs of identical pions (see [1] for a preliminary analysis). Pions produced by conventional scattering mechanisms are largely incoherent, and exhibit an intensity interference that is analogous to the Hanbury–Brown—Twiss effect for photons [2, 3]. Measurements of $\pi\pi$ correlations are typically used to study information on the size of the interaction volume in nuclear collisions, much as the HBT effect is used to measure the size of stars (although interpretation of experiments is highly nontrivial). In contrast, a fully coherent source of pions exhibits interference at the amplitude level. Consequently, the single particle spectrum from an isolated domain depends on the size of the domain, while the pair distribution is simply the product of single particle spectra.

The formation of DCC will therefore suppress the HBT effect in a nuclear collision if a large domain forms. However, the HBT effect does not completely disappear, since intensity interference can occur when a pion from the DCC has a similar momentum to a background pion. In general, the four point function determines the normalized two pion rate

$$C(p_1,p_2) \propto \sum_{i,j,k,l} \mathrm{e}^{-ip_1\cdot(x_i-x_k)-ip_2\cdot(x_j-x_l)} \langle \pi^0(x_i)\pi^0(x_j)\pi^0(x_k)\pi^0(x_l)\rangle \tag{10}$$

The HBT signal from a single DCC is

$$\begin{aligned} C(p_1,p_2) &= 1+(1-D(p_1))(1-D(p_2))\tilde{\rho}(q)^2 \\ &+ 2\{D(p_1)D(p_2)(1-D(p_1))(1-D(p_2))\}^{1/2}\tilde{\rho}(q), \end{aligned} \tag{11}$$

where $q = p_1 - p_1$ is the relative momentum of the pair and $\tilde{\rho}$ is the Fourier transform of the space–time density of the background pions. This result depends strongly on the fraction of coherent pions $D = (dN/d^3p)_{\rm DCC}/(dN/d^3p)_{\rm tot}$.

The possible impact of a DCC on $\pi^-\pi^-$ correlations is illustrated in fig. as a function of domain size, assuming that the domain produces only charged pions and that the transverse source size for the HIJING background is 7 fm. A combination of one– and two–particle measurements can allow one to disentangle the DCC signal from the background.

This estimate is derived assuming that the quantum state of the DCC is a simple type of coherent state that only conserves isospin on the average. Coherent states that strictly enforce isospin conservation have also been proposed [4]; see also the paper by Horn and Silver in ref. [2]. Kogan has proposed that the DCC may be a novel squeezed state [5]. Amado and Kogan have shown that the fluctuations of neutral and charged particles differ for these different states [5], and it is plausible that the four point function differs as well.

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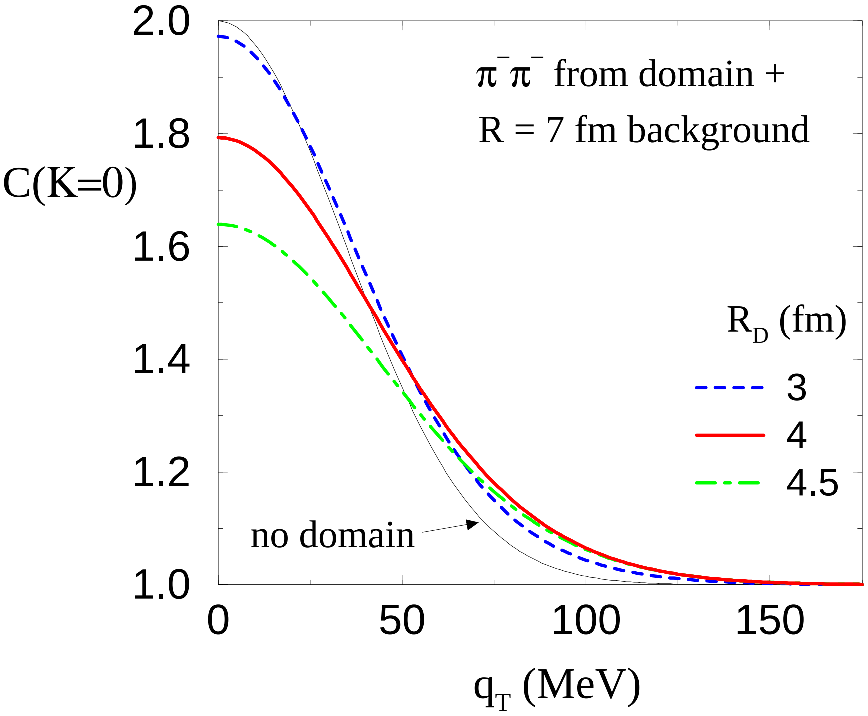

Figure 6: $\pi^-\pi^-$ correlations as a function of $q_T = p_{T1} - p_{T2}$ from a single large DCC plus a background of incoherent pions in Au+Au at RHIC. The total momentum of the pair is taken to be $K = (p_{T1} + p_{T2})/2 \equiv 0$.

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Dynamics of Chiral Condensate Formation

(Sean Gavin)

Gerd Welke and I plan to study the dynamics of DCC formation in nuclear collisions. Nuclear collisions produce a distribution of pions that, in general, bears no relation to the equilibrium distribution of pions in a "box" the size of the collision volume. Collisions and expansion then tend to drive these particles towards equilibrium. Gavin and Ruuskanen and Welke *et al.* [1] observed that SPS S+A collisions can produce pions in a larger number than the thermal equilibrium number given the energy and volume of the collision region. If a large enough number of pions were produced, a fraction of these pions would tend to form a Bose condensate as in an ideal Bose gas. Whether or not a condensate forms will depend on how rapidly the system expands compared to the time scale for condensate formation. Gavin and Ruuskanen found that the density was not high enough for condensation of an ideal Bose gas.

However, the pion fluid is not an ideal gas but a strongly interacting system, with interactions described approximately by the linear sigma model. Even if the system is in thermal thermal equilibrium at temperatures above T_c, a condensate – the sigma condensate – must form as the temperature drops. Haber and Weldon have shown that the phase diagram of the linear sigma model is very different from that of an ideal pion gas [2]. Earlier studies [1] must then be reexamined.

If the initial density is above equilibrium for that temperature and volume, then the σ and π condensate will be enriched as T drops. The time scale for condensation to occur is m_σ^{-1}, and as we know this time can be rapid in the real world where $m_\pi \neq 0$. Condensed matter physicists have made great advances in understanding the dynamics of Bose condensation. Gavin and Welke plan to apply techniques developed by Kagan et al. [3] to the study of dynamic chiral condensation at RHIC and SPS energies.

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Dilepton and Photon Production from a Coherent Pion Oscillation

(Z. Huang and X.-N. Wang)

Preliminary studies of DCC [1] have suggested that there may be a coherent pion field oscillation following a nonequilibrium second order phase transition which may lead to characteristic soft pion production [2, 3]. According to a "Baked Alaska" scenario suggested by Bjorken, Kowalski and Taylor [1], the collision debris in a high energy central event form a "hot" source on the light front and move outward while emitting quasi-goldstone bosons in the cool interior of light cone. These pions are not assumed to reach a thermal equilibrium, rather, they behave collectively and a coherent field description is more relevant. The space-time evolution of the condensate field is mainly determined by the classical equation of motion for a low energy effective theory such as a linear or nonlinear σ model. Eventually, the "disoriented vacuum" relaxes back to normal vacuum by emitting physical pions. The final hadrons, however, are emitted from on the freeze-out hyper surface and the information on the space-time evolution of the coherent field is partially lost.

In contrast, the electromagnetic signature is known to be an ideal probe of a dense hadronic matter [4] owing to the fact that it escapes the strong interaction region once produced without further final state interactions and thus carries the information on the early dynamical evolution. We study the dilepton and direct photon production from the classical pion field in the context of a disoriented chiral condensate (DCC) or most generally a nonequilibrium pion cloud. We develop a general formalism for the dilepton and photon emissions in the presence of a classical electromagnetic current. Since the electromagnetic current coincides with the third component of the isovector current, the isospin angular oscillation of the condensate field can be a significant source for the electromagnetic emission in the low mass (transverse momentum) region. As an example, we calculate the dilepton and photon spectra in an analytical solvable model where a general class of solutions are obtained by Blaizot and Krzywicki [5] and by Huang and Suzuki [6]. Although our formalism for the emission rate is quite general, the model for the classical field evolution is clearly an oversimplified version of a more realistic situation. Nevertheless, it may capture the essential features of soft pion production such as the light cone singularity, the boost invariance and the longitudinal expansion. The results obtained should only be taken as indicative, and a detailed comparison with the experimental data is only possible when a more realistic study is available. Fortunately, such an approach is already in progress [7].

The pionic part of electromagnetic interactions is introduced by replacing (we neglect the anomalous electromagnetic coupling) $\partial_\mu \boldsymbol{\pi}$ by $\partial_\mu \boldsymbol{\pi} + \frac{ie}{\sqrt{2}}[\boldsymbol{\pi}, Q]\mathcal{A}_\mu$ in the kinetic part of lagrangian

$$\mathcal{L}^{\rm em}(x) = ie[\pi^-(x)\partial_\mu \pi^+(x) - \pi^+(x)\partial_\mu \pi^-(x)]\mathcal{A}^\mu(x) = -eJ^{\rm cl}_\mu(x)\mathcal{A}^\mu(x) \ , \tag{12}$$

where $\mathcal{A}_\mu$ is the photon field and $J^{\rm cl}_\mu$ is the third component of the classical isospin current $J^{\rm cl}_\mu(x) = (\boldsymbol{\pi}(x) \times \partial_\mu \boldsymbol{\pi}(x))_3$. To $O(e^2)$, the matrix element for the dilepton emission from the classical field [8, 9, 10, 11] is

$$\mathcal{M} = \langle \ell^+\ell^- | S^{(2)} | 0 \rangle = e^2 \bar{u}(p_1)\gamma_\nu v(p_2) D^{\nu\mu}(p_1+p_2) J^{\rm cl}_\mu(p_1+p_2) \ , \tag{13}$$

where $J^{\rm cl}_\mu(p_1+p_2)$ is the Fourier transformation of $J^{\rm cl}_\mu(x)$, u and v are the Dirac spinors for the electron and positron, $D^{\nu\mu}$ is the photon propagator. The number of dileptons produced per final state phase space is

$$\begin{aligned} dN_{\ell^+\ell^-} &= \sum_{\rm spin} |\mathcal{M}|^2 \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \\ &= \int d^4q \delta^4(q-p_1-p_2) \sum_{\rm spin} |\mathcal{M}|^2 \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \ . \end{aligned} \tag{14}$$

The dilepton differential distribution with respect to the lepton pair 4-momentum q is then

$$\frac{dN_{\ell^+\ell^-}}{d^4q} = \frac{\alpha^2}{6\pi^3}\frac{B}{q^4}[q^\mu q^\nu - q^2 g^{\mu\nu}] J^{\rm cl}_\mu(q) J^{\rm cl *}_\nu(q) \ , \tag{15}$$

where $B = [1 + 2m_\ell^2/q^2][1 - 4m_\ell^2/q^2]^{1/2}$ and m_ℓ is the lepton mass. Similarly, the direct photon momentum distribution is calculated

$$\frac{dN_\gamma}{d^3\boldsymbol{q}/\omega_q} = -\frac{\alpha}{(2\pi)^2} J^{{\rm cl}\mu}(q) J^{\rm cl *}_\mu(q) \ , \tag{16}$$

where q stands for the photon 4-momentum. Moreover, splitting the current into parts parallel and orthogonal to q_μ: $J^{\rm cl}_\mu(q) = q_\mu J^{\rm para}(q) + J^{\rm tr}_\mu(q)$, one finds that only the orthogonal part contributes to the emissions. The dependence of the electromagnetic spectra on the space-time history is entirely contained in the Fourier transformation of the isospin current.

We present the idea by examining some simple scaling solutions in the 1+1 boost invariant model which are first obtained by Blaizot and Krzywicki [5]. Since the calculation is fully analytical, we hope to understand some qualitative features of dilepton production from the classical field. The lagrangian for the nonlinear σ model is

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{tr}\Big[\partial_\mu \Sigma^\dagger(x)\partial^\mu \Sigma(x)\Big] + \frac{m_\pi^2 f_\pi^2}{4}\mathrm{tr}\Big[\Sigma^\dagger(x) + \Sigma(x)\Big], \tag{17}$$

where one defines the pion field by $\sigma + i\boldsymbol{\tau}\cdot\boldsymbol{\pi} = f_\pi\Sigma$ with the constraint $\sigma = \sqrt{f_\pi^2 - \boldsymbol{\pi}^2}$. Σ transforms like $\Sigma \to U_L \Sigma U_R^\dagger$ under $SU(2)_L \times SU(2)_R$ rotations. We are interested in an idealized boost-invariant case [12] where the field $\boldsymbol{\pi} = \boldsymbol{n}(\tau)\pi(\tau, \boldsymbol{x}_\perp)$, i.e. the orientation $\boldsymbol{n}$ of the pion field is only function of the proper time τ defined by $\tau = \sqrt{t^2 - z^2}$ and uniform in spatial rapidity and transverse coordinates $\boldsymbol{x}_\perp$. This would correspond to a single DCC domain in the transverse dimension. The transverse profile of the field amplitude π^2 is determined by the shape and size of the DCC domain, which we take a Gaussian form as suggested in numerical simulations [3]

$$g(\boldsymbol{x}_\perp) = \exp[-|\boldsymbol{x}_\perp|^2/R_D^2] \ , \tag{18}$$

where R_D is the size of a DCC domain (typically $R_D = 1 \sim 2$ fm [3]). Such a class of solutions are well known to predict a distinctive neutral pion probability distribution

$$\frac{dP}{df} = \frac{1}{2\sqrt{f}} \ , \tag{19}$$

where P is the probability of find a neutral pion fraction $f = N_{\pi^0}/(N_{\pi^0} + N_{\pi^\pm})$. However, if there exists more than one DCC domain, as it is likely the case especially in heavy-ion collisions where the transverse dimension R_A of the interaction volume is much larger than the size of a DCC domain R_D, the field orientation $\boldsymbol{n}$ could depend on the transverse coordinates $\boldsymbol{x}_\perp$. In this case, the prediction (19) in general no longer holds. Such a possibility can only be studied in detail in numerical simulations. We confine ourselves to the case when the single domain dominates and only comment on how our result might change in a multi-domain case.

According to the "Baked Alaska" scenario [1], the boundary conditions for the solutions are imposed such that the classical waves propagate forward in proper time, starting at an initial time τ_0.In other words, we are interested in a retarded wave that vanishes when $\tau < \tau_0$ and $t < \tau_0$. Such a solution only exists when there is a source located on the hyperbola of equal $\tau = \tau_0$. The "hot" source can be regarded as the summation of the degrees of freedom other than pion modes. Away from the source located near the surface of the light cone, the field propagates freely and one should have the (partial) conservation of (axial) isovector currents

$$\partial_\mu \boldsymbol{V}^\mu = \partial_\mu(\boldsymbol{\pi}\times\partial^\mu\boldsymbol{\pi}) = 0 \ , \qquad (\tau > \tau_0) \tag{20}$$

$$\partial_\mu \boldsymbol{A}^\mu = \partial_\mu(\sigma\partial^\mu\boldsymbol{\pi} - \boldsymbol{\pi}\partial^\mu\sigma) = -m_\pi^2 f_\pi \boldsymbol{\pi} \ . \qquad (\tau > \tau_0) \tag{21}$$

For a function only of τ, a partial derivative $\partial_\mu f(\tau)$ is equal to $(\widetilde{x}_\mu/\tau)df/d\tau$ where $\widetilde{x}_\mu = (t, z, 0, 0)$. Eq.(20) yields the integration

$$\boldsymbol{\pi} \times \partial^\mu \boldsymbol{\pi} = f_\pi^2 \boldsymbol{a} \frac{\widetilde{x}_\mu}{\tau^2} g(\boldsymbol{x}_\perp) \ , \tag{22}$$

where $\boldsymbol{a}$ is a dimensionless constant vector in isospin space. In the chiral limit, the axial current is also conserved, which defines another constant vector $\boldsymbol{b}$ in isospin space with a constraint $\boldsymbol{a} \cdot \boldsymbol{b} = 0$ [5]

$$\boldsymbol{\pi} \partial^\mu \sigma - \sigma \partial_\mu \boldsymbol{\pi} = f_\pi^2 \boldsymbol{b} \frac{\widetilde{x}_\mu}{\tau^2} g(\boldsymbol{x}_\perp) \ . \tag{23}$$

The pion field orientation vector $\boldsymbol{n}(\tau)$ precesses with proper time τ around $\boldsymbol{a}$ in the plane perpendicular to $\boldsymbol{a}$ [6].

The dilepton and photon emissions probe the nature of the source using time-like momenta $q = (q_0, q_{||}, \boldsymbol{q}_T)$ with $q^2 = M^2$ and $q^2 = 0$ respectively. The Fourier transformation of the electromagnetic current can be readily calculated in the light cone variables

$$\begin{aligned} J_\mu^{\rm cl}(q) &= f_\pi^2 a_3 \int d^4x e^{iqx} \frac{\widetilde{x}_\mu}{\tau^2} \theta(\tau - \tau_0) g(\boldsymbol{x}_\perp) \\ &= -i f_\pi^2 a_3 \frac{\widetilde{q}_\mu}{M_\perp} \frac{\partial}{\partial M_\perp} \int_{\tau_0} \frac{d\tau}{\tau} \int d\eta e^{iM_\perp \tau \cosh(\eta - y)} \int d^2\boldsymbol{x}_\perp g(\boldsymbol{x}_\perp) e^{-i\boldsymbol{q}_T \cdot \boldsymbol{x}_\perp} \\ &= -\frac{\pi^2 R_D^2}{2} f_\pi^2 a_3 \frac{\widetilde{q}_\mu}{M_\perp^2} \exp[-q_T^2 R_D^2/4][J_0(M_\perp \tau_0) - iN_0(M_\perp \tau_0)] \ , \end{aligned} \tag{24}$$

where $\widetilde{q}_\mu = (q_0, q_{||}, 0, 0)$, $M_\perp = \sqrt{q_0^2 - q_{||}^2}$ and J_0 and N_0 are the Bessel functions. For a direct photon, $M_\perp = q_T$. In deriving (24), we have performed the τ-integration from τ_0 to infinity. Since the electromagnetic current density decreases rapidly with the proper time, the integrated current turns out to be finite and the emission is most effective in the early evolution of the classical field. It is clear from (24) that due to the singular behavior of the Neumann function for small argument, the emission spectra are sensitive to the initial time scale τ_0. The exponential factor $\exp[-q_T^2 R_D^2/4]$ provides a strong q_T cutoff so that the emission is only significant in low momentum region. The constant a_3 can be estimated using the symmetry argument. The important assumption in the disoriented chiral condensate scenario is the equal probability of all internal orientations. The energy density due to the internal chiral oscillations should be equally distributed among all components of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. One thus has

$$\langle a_i^2 \rangle = \langle b_i^2 \rangle = \frac{1}{6} \frac{\epsilon_0 \tau_0^2}{2 f_\pi^2} \qquad (i = 1, 2, 3) \ . \tag{25}$$

where ϵ_0 is the initial energy density carried by the internal chiral oscillation at $\tau = \tau_0$, which may be roughly estimated assuming that it is comparable to the thermal energy density (in order for the DCC to have any physical significance). For some typical values $\epsilon_0 \sim 0.5 T_c^4 \sim 0.1$ GeV/fm^3 and $\tau_0 = 5$ fm/c, the average value of a_3^2 is about 4.8.

In Fig. 14 we numerically integrate over $M_\perp$ and plot the dilepton invariant mass spectrum for different values of $\tau_0 = 5, 10$ fm/c and $R_D = 1, 2$ fm. The a_3 is calculated from (25) assuming that the initial energy density for the internal chiral oscillations is 0.1 GeV/fm^3. Also plotted in Fig. 14 is the typical thermal spectrum of dilepton production mainly from

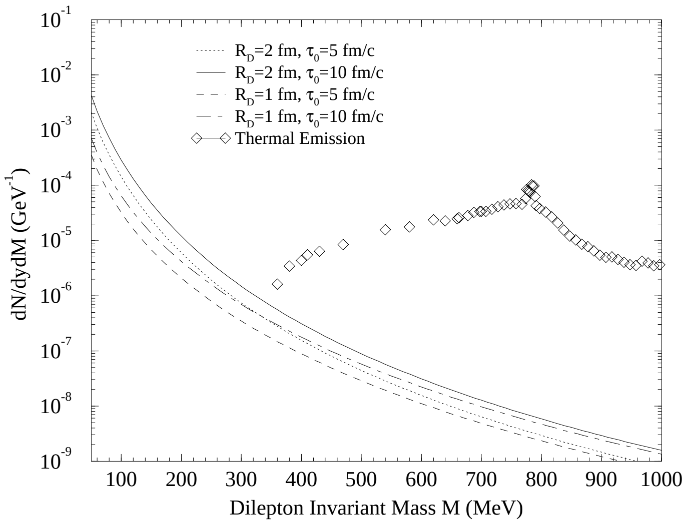

Figure 7: The dilepton invariant mass spectrum for different choices of initial time scale τ_0 and the coherent field domain size R_D. The initial energy density is assumed to be $\epsilon_0 = 0.1$ GeV/fm^3. The typical thermal spectrum due to the π-π annihilation is also plotted for a comparison.

π-π annihilation, which is taken from Ref.[11], where we have assumed the initial temperature of 160 MeV, the freeze-out temperature of 130 MeV, and the transverse size of system $R_A = 5$ fm suitable for heavy-ion collisions. Most notably, there does not exist a pion mass threshold for the dilepton production from the coherent field: the spectrum rises even below $M = 2m_\pi$. The finite pion mass plays no roles in the conserved isovector current whose third component directly couples to the photon. Therefore, the coherent field is most effective in producing lepton pairs in the low mass region $M < 2m_\pi$.

We have also studied the direct photon transverse momentum spectrum for different choices of τ_0 and R_D. The outstanding feature in the spectrum is the sensitivity on the domain size parameter R_D. The measurement of direct photon spectrum at very small transverse momentum can thus provide a sensitive probe of the spatial extension of the DCC domain.

In conclusion, we have demonstrated that the electromagnetic emission from a disoriented chiral condensate can be calculated using the space-time evolution of the classical field. The emissions due to the coherent isospin oscillation can be an important additional source to the electromagnetic signals from a dense hadronic matter, especially in low mass (momentum) region. Our results are based on an oversimplified model on classical field evolution and should only be taken as a qualitative estimation. Once a more realistic calculation is available, it would be very interesting to see whether or not the idea suggested may indeed provide some mechanism for the observed low mass dilepton enhancement by the CERES and HELIOS collaborations at CERN SPS experiments [13].

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