Collins-Soper-Sterman Approach to the Drell Yan $p_{\rm T}$ Distribution in Nuclear Collisions

Gavin P.V. Ruuskanen, D.K. Srivastava and R.L. Thews, are addressing Drell Yan $p_{\rm T}$ distributions in hadron–nucleus and nucleus–nucleus, applying techniques adapted from proton–proton collisions [1]. This work is part of the ongoing efforts of the 'hard probes collaboration' The HPC is group of theorists convened to provide benchmark estimates for hard processes at RHIC and LHC. Previously, this group provided calculations for nucleon-nucleon rates at RHIC and LHC ion-ion energies [2]. In turning to nuclear collisions, they now must consider how the nuclear target affects these rates. Possible effects that modify hadron-nucleus collisions are:

- *Initial state scattering*, which modifies transverse momentum distributions in Drell Yan [3].
- *shadowing*, which effects parton distributions in the target and, consequently, the overall rate [4].
- *energy loss*, which can effect distributions at kinematic extremes [5].
- final state scattering, which alters momentum distributions and rates of processes with hadronic final states [6].

All of these topics have been studied extensively by Gavin and Sarcevic, as reported in this proposal.

Gavin and coworkers were the first to apply phenomenological models of initial state scattering to Drell Yan and ψ production [3]. This new work is an attempt to develop a more microscopic and theoretically-justified model to confront detailed questions, such as the shapes and rapidity dependence of transverse momentum distributions at RHIC.

To begin, Gavin *et al.* will develop a microscopic formulation of elastic initial state scattering in the Drell Yan process in hadron-nucleus collisions. This formulation is similar in spirit to an earlier one developed by Chiappetta and Pirner [7]. The novel feature of the new formulation is that it incorporates the empirical result that initial state scattering is essentially a soft process, while the Drell Yan process is hard. The distinction between soft, resummed and hard contributions to the Drell Yan transverse momentum distribution is clearest in the Collins-Soper-Sterman formulation [12].

The transverse momentum dependence of the Drell Yan process has always been of interest in nucleon-nucleon collisions. Experiments show that the net transverse momenta of lepton pairs produced by the Drell-Yan process are of the order of 1 GeV for a dimuon mass $M = 10$ GeV. Such values are substantially smaller than the transverse momenta $\sim M/2$ carried by each of the leptons individually. On the other hand, the $p_{\rm T}$ of a Drell-Yan pair is much larger than the few-hundred MeV typical of soft QCD. If we neglect the transverse momentum of the incoming partons, then the lowest order process $q\bar{q} \rightarrow \gamma^* \rightarrow e^+e^-$ produces a final state with net $p_{\rm T} = 0$. While any spread in the initial momentum will increase the final $p_{\rm T}$ on average, the intrinsic width of the parton distribution is rather small,

$\langle p_{\rm T}^2\rangle_{\rm soft} \sim (0.3\ {\rm GeV})^2$. This scale is determined by the inverse hadron size, since the target and projectile partons cannot be localized within their parent hadrons. Therefore, we can attribute part of the measured $p_{\rm T}$ to the parton's intrinsic $p_{\rm T}$, but not all.

The dimuon pair acquires additional transverse momentum from production mechanisms that occur beyond leading order in perturbation theory [8, 9]. For example, the Compton and annihilation processes

$$qg \to q\gamma^* \quad \text{and} \quad \bar{q}q \to \gamma^* g \tag{1}$$

occur in which the dimuon's $p_{\rm T}$ can be balanced by the recoil of the final state quark or gluon. One can compute the $p_{\rm T}$ distribution perturbatively from these processes and their radiative corrections. The perturbation expansion is well behaved for $p_{\rm T} \sim M$. However, at low $p_{\rm T}$ the perturbation expansion breaks down and a resummation of the perturbation series is required.

To see why this resummation is necessary, observe that the cross section in the region $p_{\rm T}^2 << M^2$ is dominated by the leading-logarithm contributions:

$$\frac{{\rm d}\sigma}{{\rm d}p_{\rm T}^2} \sim \frac{\alpha_{\rm W}\alpha_{\rm s}}{p_{\rm T}^2} \ln\left(\frac{Q^2}{p_{\rm T}^2}\right)\left[v_1 + v_2\alpha_{\rm s}\ln^2\left(\frac{Q^2}{p_{\rm T}^2}\right) + v_3\alpha_{\rm s}^2\ln^4\left(\frac{Q^2}{p_{\rm T}^2}\right) + \cdots\right], \tag{2}$$

where $\alpha_{\rm s}$ is evaluated at the scale Q^2. This series is effectively an expansion in $\alpha_{\rm s}\ln^2(Q^2/p_{\rm T}^2)$, rather than $\alpha_{\rm s}$ alone. The effective expansion parameter can be large at low $p_{\rm T}$ even if $\alpha_{\rm s}(Q^2)$ is small. The leading-logarithm series (2) describes the effect of soft gluon radiation from the initial state q and $\bar{q}$ prior to their annihilation. Specifically, these logarithms are remnants of the mass and collinear singularities arising from the radiated gluons. The annihilation process in (1) contributes the term $\propto \alpha_{\rm s}\ln(Q^2/p_{\rm T}^2)/p_{\rm T}^2$ and, in general, $q\bar{q} \to \gamma^* + n$ gluons produces the term of order $\alpha_{\rm s}^n$. Fortunately, the coefficients v_i of Eq. (2) are not independent and it is possible to sum the series exactly so that it applies even when $\alpha_{\rm s}\ln^2(Q^2/p_{\rm T}^2)$ is large.[10, 11, 12] In addition, 'subleading' logarithm contributions, though smaller, can also be important.

The formalism needed to sum the leading and subleading logarithms was developed by [12] by Collins, Soper and Sterman. For each species of colliding partons, one finds

$$\frac{{\rm d}\sigma}{{\rm d}p_{\rm T}^2\,{\rm d}y}({\rm resum}) = \frac{4\pi^3\alpha_{\rm W}}{3s}\int \frac{{\rm d}^2 b}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{p}_{\rm T}} W(b), \tag{3}$$

$$\begin{aligned} W(b) \;&=\; \exp\left\{-\int_{\beta/b^2}^{Q^2}\frac{{\rm d}q^2}{q^2}\left[\ln\left(\frac{Q^2}{q^2}\right)A(\alpha_{\rm s}(q^2)) + B(\alpha_{\rm s}(q^2))\right]\right\} \\ &\quad \times\ (C\circ f_A)(x_A;\beta^2/b^2)\ \times\ (C\circ f_B)(x_B;\beta^2/b^2). \end{aligned} \tag{4}$$

where s is the total hadronic center-of-mass energy, $f_{\rm A}$ and $f_{\rm B}$ are the parton structure functions of the two colliding particles, and

$$x_{\rm A} = e^{+y}\sqrt{Q^2/s}, \qquad x_{\rm B} = e^{-y}\sqrt{Q^2/s} \tag{5}$$

are the dominant values of x as $p_{\rm T} \to 0$. The integration variable b is the impact parameter, the variable conjugate to $p_{\rm T}$, and $\beta \equiv 2\,{\rm e}^{-\gamma_{\rm E}}$, where $\gamma_{\rm E}$ is Euler's constant. The function C is a coefficient function that converts the structure functions f into distributions $C \circ f$ specific to the process at hand. The functions A, B, and $C(x)$ have perturbative expansions in $\alpha_{\rm s}$, with A and B starting at order $\alpha_{\rm s}$. The expansion for C begins at order 1 for quarks and order $\alpha_{\rm s}$ for gluons. These functions can be extracted to a given order from the perturbative result, and have been determined for the Drell Yan process at next-to-leading order by Davies *et al.*

The resummed result (3) applies only when $p_{\rm T}^2 \ll Q^2$ because it includes only those terms that diverge as $1/p_{\rm T}^2$ as $p_{\rm T} \to 0$. The nonsingular contributions to the $p_{\rm T}$ distribution omitted in (3) become important at large $p_{\rm T}$ because they fall off less rapidly. On the other hand, conventional perturbation theory works well at large $p_{\rm T}$, describing the complete $p_{\rm T}$ dependence to a given order in $\alpha_{\rm s}$. Bridging the two regions is accomplished by adding in the terms that are not resummed, the so-called remainder or non-singular terms. Arnold and Kauffman developed a prescription for calculating the remainder terms that explicitly matches the high and low $p_{\rm T}$ results.

The Arnold and Kauffman prescription proceeds as follows. One first expands the resummed result (3) in powers of $\alpha_{\rm s}$. This series, ${\rm d}\sigma/{\rm d}p_{\rm T}^2\,{\rm d}y({\rm asym})$, contains the singular 'asymptotic' $1/p_{\rm T}^2$ part of complete perturbation series ${\rm d}\sigma/{\rm d}p_{\rm T}^2\,{\rm d}y({\rm pert})$. The asymptotic result in Ref. [16] is expressed as convolutions of structure functions with the coefficient functions of (3) and with Altarelli-Parisi splitting functions arising from the scale dependence of the structure functions. With the singular terms isolated in the asymptotic result, the remainder is the difference between the perturbative result and the asymptotic result, $R = {\rm d}\sigma/{\rm d}p_{\rm T}^2\,{\rm d}y({\rm pert}) - {\rm d}\sigma/{\rm d}p_{\rm T}^2\,{\rm d}y({\rm asym})$. The perturbation series for the $p_{\rm T}$ distribution — and therefore R — has been computed to 2nd order Ref. [9]. The total cross section is then written

$$\frac{{\rm d}\sigma}{{\rm d}p_{\rm T}^2\,{\rm d}y}({\rm total}) = \frac{{\rm d}\sigma}{{\rm d}p_{\rm T}^2\,{\rm d}y}({\rm resum}) + \frac{{\rm d}\sigma}{{\rm d}p_{\rm T}^2\,{\rm d}y}({\rm pert}) - \frac{{\rm d}\sigma}{{\rm d}p_{\rm T}^2\,{\rm d}y}({\rm asym}). \tag{6}$$

The "matching" is now manifest: at low $p_{\rm T}$ the perturbative and asymptotic pieces cancel, leaving the resummed; at high $p_{\rm T}$ the resummed and asymptotic pieces cancel to 2nd order, leaving the perturbative contribution. The relative error is explicitly of order $\alpha_{\rm s}^2$, see Ref. [16].

At very high $p_{\rm T}$ the matching prescription breaks down and one must switch back to the perturbative result. This breakdown occurs because R is only known to 2nd order, while ${\rm d}\sigma/{\rm d}p_{\rm T}^2\,{\rm d}y({\rm resum})$ in effect contains all orders in $\alpha_{\rm s}$. For example, ${\rm d}\sigma/{\rm d}p_{\rm T}^2\,{\rm d}y({\rm resum})$ introduces terms $\propto \alpha_{\rm s}^3(\ln p_{\rm T})^5/p_{\rm T}^2$ that will not be canceled in the 2nd order expression for R. Such terms are important at large $p_{\rm T}$ when ${\rm d}\sigma/{\rm d}p_{\rm T}^2\,{\rm d}y$ has fallen off to the extent that R is comparable to the total. From that point on, the perturbative prediction is more reliable than (6). The switch is done at sufficiently high $p_{\rm T}$ so that the error incurred is free of large logarithms.

The form factor $W(b)$ contains $\alpha_{\rm s}$ and parton distributions evaluated at the scale $1/b$, and its evaluation is problematic for $b > 1$ GeV^{-1}. Moreover, one wishes to include the effect of the intrinsic $p_{\rm T}$ of the partons. Both of these ends are met by replacing

$$W(b) \to W(b_*){\rm e}^{-S_{\rm np}(b)} \tag{7}$$

where $b_* = b/\sqrt{1 + (b/b_{\rm max})^2}$ and $b_{\rm max} = 0.5$ GeV^{-1}. The Collins-Soper-Sterman formalism specifies that $S_{\rm np}$ have a term which depends on $\ln Q$ and a term which does not and that the

$\ln Q$ term does not depend on the colliding hadrons or on the parton x's. However, beyond these constraints $S_{\rm np}$ is arbitrary and must be extracted from experiment. Ladinsky and Yuan parametrize

$$S_{\rm np} = g_1 b\,[b + g_3 \ln(\tau/\tau_1)] + g_2 b^2 \ln(Q/2Q_1), \tag{8}$$

where $\tau = x_A x_B$ [17]. To fit the ISR $p_{\rm T}$ distribution from R209, they take $g_1 = 0.11$ GeV^2, $g_2 = 0.58$ GeV^2, $g_3 = -1.5$ GeV^2, $\tau_1 = 0.01$ and $Q_1 = 1.6$ GeV. Note that these parameter choices are somewhat different from those in Ref. [15, 16].

Momentum distributions presented in [1] were computed using a code adapted from Ref. [16]. Values of the nonperturbative parameters in $S_{\rm np}$ were taken from earlier studies of $W^\pm$ and Z production at high Q^2. Agreement with Drell Yan data at much lower Q^2 was fair. As a first step in our new work, we intend to refit the nonperturbative parameters in $S_{\rm np}$.

Data suggests that multiple initial state parton scattering increases the square of the transverse momentum in accord with

$$\langle p_{\rm T}^2\rangle_{pA} = \langle p_{\rm T}^2\rangle_{pp} + \kappa^2(\langle n\rangle - 1)/2, \tag{9}$$

where $\langle n\rangle$ is the average number of initial state scatterings. Observe that recent Drell Yan data imply that the momentum transferred in each initial state scattering, κ, roughly is on the order of soft scales, ~ 200 MeV [18]. In the Collins-Soper-Sterman formalism, the effect of soft scales is parametrized by the nonperturbative form factor $S_{\rm np}$. In describing hA interactions, Gavin *et al.* introduce a form factor $S_{\rm np}(n)$ that depends on the number of scatterings and reproduces the random walk formula. The hA cross section is then:

$$\left(\frac{d\sigma}{dp_{\rm T}}\right)_{pA} = \sum_n \int d^2b\,dz\,\rho(b,z) P_n^i(b,z) \left(\frac{d\sigma}{dp_{\rm T}}\right)(n), \tag{10}$$

where P_n^i is the Glauber probability for a NN collision prior to the hard Drell Yan process, and $(d\sigma/dp_{\rm T})(n)$ is the same as in pp collisions but with $S_{\rm np}$ replaced by $S_{\rm np}(n)$. This approach is formally similar to ref. [7], except that, here, initial state scattering is taken to contribute to the soft part of S, while in [7] it contributes to the resummed part. In addition, experimental knowledge of Drell Yan in pA collisions has improved considerably since [7]. First results will be reported to the hard probes collaboration at the next meeting at INT/Seattle this spring.

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