CRITICAL ACCELERATION

Elementary interactions under extreme conditions

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I discuss foundational ultra-high acceleration physics challenges arising in the context of strong field physics realized in several areas of physics. This will lead us to the exploration of the quantum vacuum the modern day aether. We will recognize the need to reconsider formulation of laws governing the basic forces in order to include the physics of critical acceleration $a/m = c^3/\hbar$. Connection to strong field particle production, Mach Principle, Unruh and Hawking radiation will be discussed.

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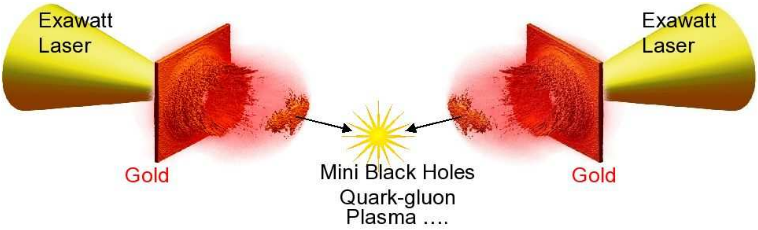
Overview

1 Critical Acceleration: Electron-Laser and RHI Collisions
2 Challenges to foundational understanding: Laws of Physics, Inertia: Mach, Aether and Quantum Vacuum
3 EM Interaction and Radiation Reaction
4 Quantum Physics: QED of Strong Fields



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Critical Acceleration

An electron in presence of the critical 'Schwinger' (Vacuum Instability) field strength of magnitude:

$$E_s = \frac{m_e^2 c^3}{e\hbar} = 1.323 \times 10^{18} V/m$$

is subject to critical natural unit =1 acceleration:

$$a_c = \frac{m_e c^3}{\hbar} \to 2.331 \times 10^{29} \mathrm{m/s}^2$$

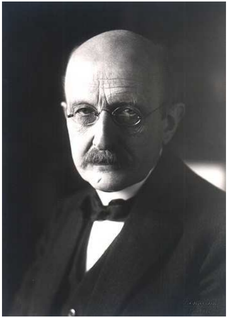
Truly dimensionless unit acceleration arises when we introduce specific acceleration

$$\aleph = \frac{a_c}{mc^2} = \frac{c}{\hbar}$$

Specific unit acceleration arises in Newton gravity at Planck length distance: $\aleph_G \equiv G/L_p^2 = c/\hbar$ at $L_p = \sqrt{\hbar G/c}$.

In the presence of sufficiently strong electric field E_s by virtue of the equivalence principle, electrons are subject to Planck 'critical' force.

1899:
Planck units



$$h/k_B = a = 0.4818\cdot10^{-10}\,[\text{sec}\times\text{Celsiusgrad}]$$
$$h = b = 6.885\cdot10^{-27}\left[\frac{\text{cm}^2\,\text{gr}}{\text{sec}}\right]$$
$$c = c = 3.00\cdot10^{10}\left[\frac{\text{cm}}{\text{sec}}\right]$$
$$G = f = 6.685\cdot10^{-8}\left[\frac{\text{cm}^3}{\text{gr.}\,\text{sec}^2}\right]^1.$$

Wählt man nun die »natürlichen Einheiten« so, dass in dem neuen Maasssystem jede der vorstehenden vier Constanten den Werth 1 annimmt, so erhält man als Einheit der Länge die Grösse:

$$\sqrt{2\pi}\,L_{Pl} = \sqrt{\frac{bf}{c^3}} = 4.13\cdot10^{-33}\,\text{cm}, \mapsto \sqrt{2\pi}\,1.62\times10^{-33}\ \text{cm}$$

als Einheit der Masse:

$$\sqrt{2\pi}\,M_{Pl} = \sqrt{\frac{bc}{f}} = 5.56\cdot10^{-5}\,\text{gr}, \mapsto \sqrt{2\pi}\,2.18\times10^{-5}\ \text{g}$$

als Einheit der Zeit:

$$\sqrt{2\pi}\,t_{Pl} = \sqrt{\frac{bf}{c^5}} = 1.38\cdot10^{-43}\,\text{sec}, \mapsto \sqrt{2\pi}\,5.40\times10^{-44}\ \text{s}$$

als Einheit der Temperatur:

$$\sqrt{2\pi}\,T_{Pl} = a\sqrt{\frac{c^5}{bf}} = 3.50\cdot10^{32}\,{}^\circ\text{Cels.} \mapsto \sqrt{2\pi}\,1.42\times10^{32}\ \text{K}$$

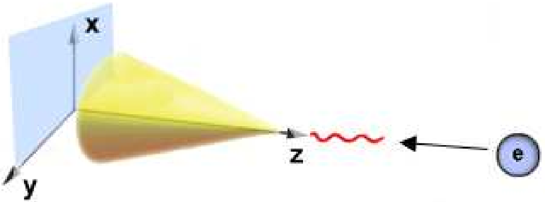
Diese Grössen behalten ihre natürliche Bedeutung so lange bei, als die Gesetze der Gravitation, der Lichtfortpflanzung im Vacuum und die beiden Hauptsätze der Wärmetheorie in Gültigkeit bleiben, sie müssen also, von den verschiedensten Intelligenzen nach den verschiedensten Methoden gemessen, sich immer wieder als die nämlichen ergeben.

"These scales retain their natural meaning as long as the law of gravitation, the velocity of light in vacuum and the central equations of thermodynamics remain valid, and therefore they must always arise, among different intelligences employing different means of measuring." **M. Planck,** *"Über irreversible Strahlungsvorgänge." Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin* **5**, *440-480 (1899),* **(added on last page)**

Probing super-critical (Planck) acceleration

$$a_c = 1(\rightarrow m_e c^3/\hbar = 2.331 \times 10^{29}\mathrm{m/s}^2)$$

Plan A: Directly laser accelerate electrons from rest, requires Schwinger scale field and may not be realizable – backreaction and far beyond today's laser pulse intensity technology.
Plan B: Ultra-relativistic Lorentz-boost: we collide counter-propagating electron and laser pulse.



Solution with radiation reaction showing critical acceleration:
Effects of Radiation-Reaction in Relativistic Laser Acceleration
Y. Hadad, et al, Phys.Rev. D **82**, 096012 (2010)

Laser pulse in electron's rest frame

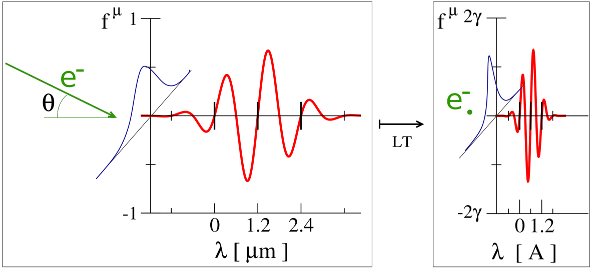


Figure shows boost (from left to right) of the force applied by a Gaussian photon pulse to an electron, on left counter propagating with $\gamma/\cos\theta = 2000$. Pulse narrowed by $(\gamma\cos\theta)^{-1}$ in the longitudinal and $(\gamma\sin\theta)^{-1}$ in the transverse direction. Corresponds to Doppler-shift:

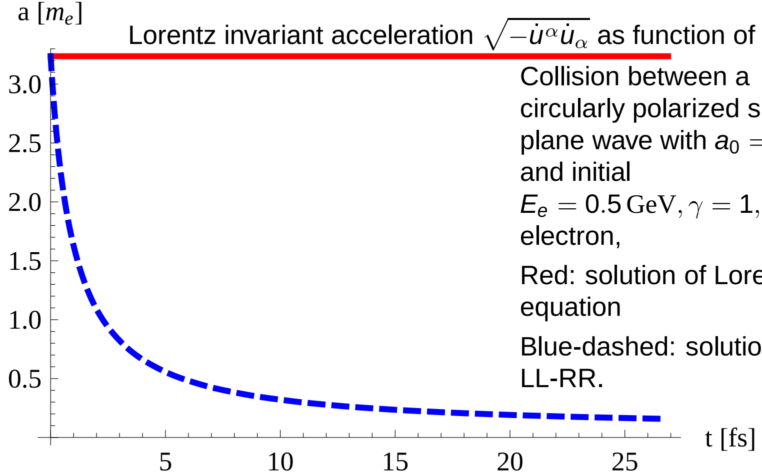
$$\omega \rightarrow \omega' = \gamma(\omega + \vec{v} \cdot nk)$$

as applied to different frequencies making up the pulse.

Example: Electron de-acceleration by a pulse

Y. Hadad, et al, Phys.Rev.D 82, 096012 (2010)

Lorentz invariant acceleration $\sqrt{-\dot{u}^{\alpha}\dot{u}_{\alpha}}$ as function of time



Collision between a circularly polarized square plane wave with $a_0 = 100$ and initial $E_e = 0.5\,\mathrm{GeV}, \gamma = 1,000$ electron,

Red: solution of Lorentz equation

Blue-dashed: solution of LL-RR.

Studied other interesting cases, of which electron scattering on pulses most promising.

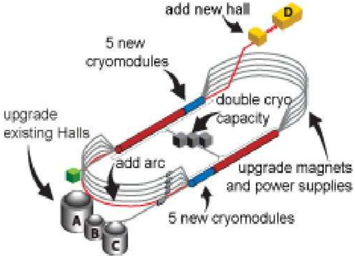
Experiments probing critical acceleration possible

experiments are feasible today

at SLAC, and at CEBAF: There is a 12 GeV ($\gamma = 2400$) electron beam

There is a laser team

There is appropriate high radiation shielded experimental hall

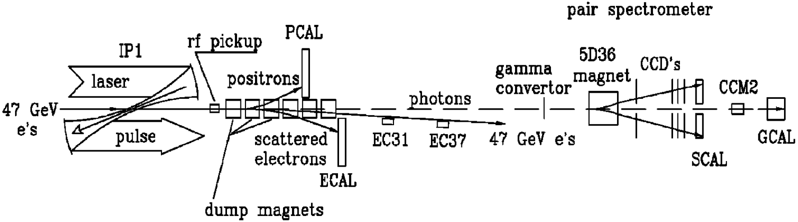


SLAC'95 experiment below critical acceleration

$p_e^0 = 46.6$ GeV; in 1996/7 $a_0 = 0.4$, $\left|\frac{du^\alpha}{d\tau}\right| = .073[m_e]$ (Peak)

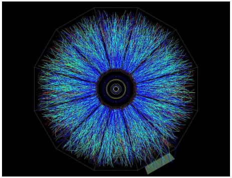
Multi-photon processes observed:

- Nonlinear Compton scattering
- Breit-Wheeler electron-positron pairs



- **D. L. Burke** *et al.*, "Positron production in multiphoton light-by-light scattering," Phys. Rev. Lett. **79**, 1626 (1997)
- **C. Bamber** *et al.*, "Studies of nonlinear QED in collisions of 46.6 GeV electrons with intense laser pulses" Phys. Rev. D **60**, 092004 (1999).

Critical acceleration probably achieved at RHIC



Two nuclei smashed into each other from two sides: components 'partons' can be stopped in CM frame within $\Delta\tau \simeq 1$ fm/c. Tracks show multitude of particles produced, as observed at RHIC (BNL).

• The acceleration a achieved to stop some/any of the components of the colliding nuclei in CM: $a \simeq \frac{\Delta y}{M_i \Delta\tau}$. Full stopping: $\Delta y_{\rm SPS} = 2.9$, and $\Delta y_{\rm RHIC} = 5.4$. Considering constituent quark masses $M_i \simeq M_N/3 \simeq 310$ MeV we need $\Delta\tau_{\rm SPS} < 1.8$ fm/c and $\Delta\tau_{\rm RHIC} < 3.4$ fm/c to exceed a_c.

• Observed unexplained soft electromagnetic radiation in hadron reactions *A. Belognni et al. [WA91 Collaboration], "Confirmation of a soft photon signal in excess of QED expectations in* π*–p interactions at 280-GeV/c," Phys. Lett. B* **408**, *487 (1997)*

. • Recent suggestions that thermal hadron radiation due to Unruh type phenomena *P. Castorina, D. Kharzeev and H. Satz, "Thermal Hadronization and Hawking-Unruh Radiation in QCD," Eur. Phys. J. C* **52**, *187 (2007), also Biro, Gyulassy [arXiv:1111.4817]*

Why bother??

We do not understand the laws of physics near and beyond critical acceleration.

- Force capable of critical acceleration incompatible with present view of vacuum
- Charged particle dynamics altered decisively by radiation reaction

• The description of physical reality based on an action integral derived from daily experience, subject to assumed symmetry constraints, which imply assumption of conservation laws according to Noether's theorem. In that sense all forces are effective.

• The specific form of the force-law we investigate is outcome of some limiting process applied to yet more foundational not yet discovered context. This description is considered successful if and when the particle dynamics is for most part accounted by.

• This is not the case if higher order corrections such as back reaction must be introduced to understand even in principle particle dynamics in supercritical fields.

Example: we are sure that the Lorentz Force in presence of critical fields is incomplete.

The 21st Century Foundational Physics Challenges

- How does the structured quantum vacuum control inertia, and many other laws of physics
- Is there a deeper understanding of time?
  Rôle of the Universe expansion in defining time?
- What is the cosmological dark energy,

$$\lambda = (2.4\mathrm{meV})^4 \hbar^{-3} c^{-5} = 4.3\mathrm{keV} c^{-2}/\mathrm{cm}^3$$

  Is this excited state quantum Vacuum with non-zero point energy? If so how can we induce vacuum decay? What is the magnitude of causal velocity ('velocity of light') in the different vacuum states?
- What is the origin of grand scales as expressed by
  a) the Planck mass: $M_P = \sqrt{\hbar c / G_N} = 1.3\,10^{19} m_{\rm proton}$ and
  b) Higgs VEV $\langle h \rangle = 254\ \mathrm{GeV} \simeq 10^{13} m_{\rm neutrino}$.
- Space-time: dimensionality (3+1) $\rightarrow$ $(n+1)$: $n > 3$?
  Fractal dimension $n < 3$? Lattice? (Mem)brane?

Discovery is driven by new tools: acceleration

Challenge 1: Mach and Acceleration

To measure acceleration we need to refer to an inertial frame of reference. Once we know one inertial observer, the class of inertial observers is defined.

Before Einstein's relativity, Mach posited, *I paraphrase/simplify* as an alternative to Newton's absolute space, a) that inertia (resistance to acceleration) could be due to the background mass in the Universe, and b) that the reference inertial frame must be inertial with respect to the Universe mass rest frame. The latter postulate Einstein called 'Mach's principle'.

- General Relativity and GR based Cosmology motivated by Mach $\rightarrow$ Einstein's æther
- Any observer inertial wrt CMB frame qualifies – present day "Mach's fixed stars" frame of reference;
- QFT provides Mach's inertial frame: the quantum vacuum= æther
- Corollary: Inertia, the resistance to acceleration requires presence of non-geometric forces and/or extended quantum matter leading to rigid extended material body.

Side remark: Einstein's Aether Reference Frame

Albert Einstein at first rejected æther as unobservable when formulating special relativity, but eventually changed his initial position, re-introducing what is referred to as the 'relativistically invariant' æther. In a letter to H.A. Lorentz of November 15, 1919, see page 2 in *Einstein and the Æther*, L. Kostro, Apeiron, Montreal (2000). he writes: *It would have been more correct if I had limited myself, in my earlier publications, to emphasizing only the non-existence of an æther velocity, instead of arguing the total non-existence of the æther, for I can see that with the word æther we say nothing else than that space has to be viewed as a carrier of physical qualities.*

In a lecture published in May 1920 (given on 27 October 1920 at Reichs-Universität zu Leiden, addressing H. Lorentz), published in Berlin by Julius Springer, 1920, also in Einstein collected works: In conclusion:

. . . space is endowed with physical qualities; in this sense, therefore, there exists an æther. . . . But this æther may not be thought of as endowed with the quality characteristic of ponderable media, as (NOT) consisting of parts which may be tracked through time. The idea of motion may not be applied to it.

Challenge 2:
Electromagnetic mass and pair production

Return to the quest, the origin of mass: could the electromagnetic energy in the field of a point particle explain its mass? Born-Infeld nonlinear electromagnetism proposed around 1935 in analogy to limiting velocity concept:

$$\mathcal{L} = \frac{E^2}{2} \to \frac{E_{\rm limit}^2}{2}\sqrt{1-\frac{E^2}{E_{\rm limit}^2}}, \quad E_{\rm limit} \equiv \frac{e}{r_{\rm limit}^2} \quad , \boxed{M_e = k\frac{e^2}{r_{\rm limit}}}$$

There is limiting field strength and thus an inadvertent outcome in context of later (1950) arising problem of vacuum instability: integrable field energy in context of a Born-Infeld nonlinear limiting field theory also can assure vacuum stability. However, note: $E_s = e/\lambda\!\!\!^{-}_e{}^2$, $E_{\rm limit} = e/r_{\rm clas}^2$ Note the connection:
vacuum instability↔limiting field strength↔mass/inertia

Challenge 3: Radiation Reaction

Maxwell, Lorentz and Inertia: ad-hoc combination

The action $\mathcal{I}$ comprises three elements:

$$\mathcal{I} = -\frac{1}{4}\int d^4x\, F^2 + q\int_{\rm path} d\tau u\cdot A + \frac{mc}{2}\int_{\rm path} d\tau(u^2-1).,$$

$F^{\alpha\beta} \equiv \partial^\alpha A^\beta - \partial^\beta A^\alpha, u^\alpha = ds^\alpha/d\tau$. Path is fixed at the end points assuring gauge-invariance of dynamic equations:

• The two first terms upon variation with respect to the A-field, produce **Maxwell equation**, with sources given by motion of charged particles along prescribed world lines. Note, these are allowing radiation emission in presence of accelerated charges.

• The second and third term, when varied with respect to the shape of the material particle world line, produce the **Lorentz force** equation = particle dynamics in presence of a prescribed A-field.

Maxwell, Lorentz and Inertia: Consistency??

1) Maxwell Equation: obtain fields $F^{\beta\alpha}$ including radiation fields from a given source of fields $j^{\alpha}(x) = \sum_i \int d\tau_i u_i^{\alpha} q_i \delta^4(x - s_i(\tau_i))$

$$\partial_\beta F^{*\,\beta\alpha} = 0, \qquad \partial_\beta F^{\beta\alpha} = j^\alpha \quad \to F^{\beta\alpha}(x)$$

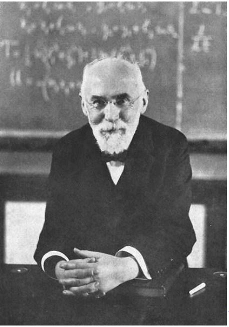
2) Inertial Force = Lorentz-force –> using fields we obtain world line of particles needed in step **1)** (Vlasov: in phase space)

$$m_e \frac{du^\alpha}{d\tau} = -eF^{\alpha\beta}u_\beta \quad \to s^\alpha(\tau),\ \frac{ds^\alpha}{d\tau} \equiv u^\alpha(\tau) \quad \to j^\alpha$$

Problem: Fields accelerate charges, accelerated charges radiate, but that alters field present in the Lorentz equation. We did it wrong, since the solution of Lorentz equation should already know of motion in the radiation field generated. The lack of consistency is recognized realizing that Lorentz equation is conservative, but radiation removes energy from particle motion.

As long as acceleration is small, radiation emitted can be incorporated as a perturbative iterative additional force.

Radiation reaction Lorentz-Abraham-Dirac (LAD) force



energy-momentum radiated:

$$\frac{dp^\alpha}{d\tau} = u_\beta q(F^{\beta\alpha} + F^{\beta\alpha}_{\rm rad}),$$

$$F^{\beta\alpha}_{\rm rad} = \frac{2q}{3}(\ddot{u}^\beta u^\alpha - \ddot{u}^\alpha u^\beta)$$

Recognized and (further) developed among others by

← Lorentz Dirac →

<u>At critical acceleration</u> the radiation reaction has same magnitude as the field force.

Two NEW Problems

The mass problem:
The appearance of electromagnetic mass next to inertial mass.
Finite EM mass can be achieved within nonlinear EM of Born-Infeld.
But, given the smallness of electron mass even if all mass is of EM origin experiments are in disagreement with this theory.

The self-acceleration and the causality problem:
The appearance of a third derivative $\ddot{u}^\alpha,\ u^\alpha = \dot{x}^\alpha$ requires assumption of an additional boundary condition to arrive at a unique solution describing the motion of a particle. Only a boundary condition in the (infinite) future allows to eliminate self-accelerating charges, that is 'run-away' solutions. Such a constraint is in conflict with the principle of causality.

This LAD radiation reaction description is universally rejected. A theoretical cure is not known, a cottage industry of ad-hoc modifications of radiation-reaction dynamics arose.

Sample of proposed LAD extensions

LAD	$\mathbf{m}\dot{\mathbf{u}}^{\alpha} = \mathbf{q}\mathbf{F}^{\alpha\beta}\mathbf{u}_{\beta} + m\tau_0 \left[\ddot{u}^{\alpha} + u^{\beta}\ddot{u}_{\beta}u^{\alpha}\right]$
Landau-Lifshitz	$\mathbf{m}\dot{\mathbf{u}}^{\alpha} = \mathbf{q}\mathbf{F}^{\alpha\beta}\mathbf{u}_{\beta} + q\tau_0 \left\{F^{\alpha\beta}_{,\gamma}u_{\beta}u^{\gamma} + \frac{q}{m}\left[F^{\alpha\beta}F_{\beta\gamma}u^{\gamma} - (u_{\gamma}F^{\gamma\beta})(F_{\beta\delta}u^{\delta})u^{\alpha}\right]\right\}$
Caldirola	$\mathbf{0} = \mathbf{q}\mathbf{F}^{\alpha\beta}(\tau)\,\mathbf{u}_{\beta}(\tau) + \frac{m}{2\tau_0}\left[u^{\alpha}(\tau - 2\tau_0) - u^{\alpha}(\tau)\,u_{\beta}(\tau)\,u^{\beta}(\tau - 2\tau_0)\right]$
Mo-Papas	$\mathbf{m}\dot{\mathbf{u}}^{\alpha} = \mathbf{q}\mathbf{F}^{\alpha\beta}\mathbf{u}_{\beta} + q\tau_0\left[F^{\alpha\beta}\dot{u}_{\beta} + F^{\beta\gamma}\dot{u}_{\beta}u_{\gamma}u^{\alpha}\right]$
Eliezer	$\mathbf{m}\dot{\mathbf{u}}^{\alpha} = \mathbf{q}\mathbf{F}^{\alpha\beta}\mathbf{u}_{\beta} + q\tau_0\left[F^{\alpha\beta}_{,\gamma}u_{\beta}u^{\gamma} + F^{\alpha\beta}\dot{u}_{\beta} - F^{\beta\gamma}u_{\beta}\dot{u}_{\gamma}u^{\alpha}\right]$
Caldirola-Yaghjian	$\mathbf{m}\dot{\mathbf{u}}^{\alpha} = \mathbf{q}\mathbf{F}^{\alpha\beta}(\tau)\,\mathbf{u}_{\beta}(\tau) + \frac{m}{\tau_0}\left[u^{\alpha}(\tau - \tau_0) - u^{\alpha}(\tau)\,u_{\beta}(\tau)\,u^{\beta}(\tau - \tau_0)\right]$

P. A. M. Dirac, “Classical theory of radiating electrons,” Proc. Roy. Soc. Lond. A **167**, 148 (1938)
L. D. Landau and E. M. Lifshitz, “The Classical theory of Fields,” *Oxford: Pergamon (1962)* 354p.
P. Caldirola, “A Relativistic Theory of the Classical Electron,” Riv. Nuovo Cim. **2N13**, 1 (1979).
T. C. Mo and C. H. Papas, “A New Equation Of Motion For Classical Charged Particles,” Izv. Akad. Nauk Arm. SSR Fiz. **5**, 402 (1970)
C. Eliezer, “On the classical theory of particles” Proc. Roy. Soc. Lond. A **194**, 543 (1948).
A. D. Yaghjian, “Relativistic Dynamics of a Charged Sphere,” *Lecture Notes in Physics, Springer-Verlag, Berlin (1992) 152p.*

Other recent references

H. Spohn, *Dynamics of charged particles and their radiation field,* (CUP, Cambridge, UK 2004, ISBN 0521836972)
F. Rohrlich, “Dynamics of a charged particle” Phys. Rev. E **77**, 046609 (2008)

Insight: EM interaction at Critical Acceleration

Maxwell and Lorentz equations ARE INCONSISTENT, present day action respects gauge- and relativistic-invariance but modifications are possible and required, a few were tried, a solution was not found:

- Born-Infeld theory introduces nonlinear field action aiming to limit the achievable field strength, and thus to limit acceleration. Problem: Field needed to explain electron mass too high.
- Caldirola proposed discrete action generalization of Lorentz particle dynamics in an attempt to address the radiation-reaction inconsistency, dynamics accommodates electron mass but time quantized in manner inconsistent with experiment.
- I am not aware of a EM theory formulation that addresses the need to relate acceleration to a background Mach's inertial frame.
- QED is build on same ideas and beset by similar problems in the limit of strong fields = critical acceleration. Example: Thompson scattering a classical back reaction problem but also long wave-length limit of Compton quantum scattering.

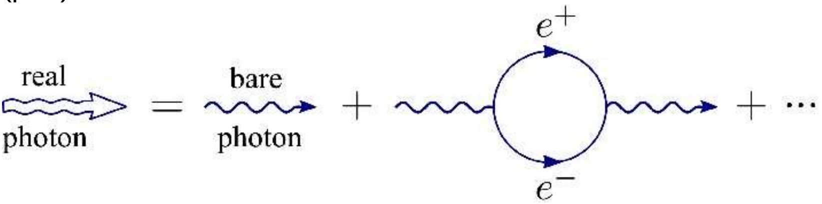
Importance of Quantum Theory

Without quantum theory we would not have extended bodies, without extended material bodies we cannot create critical acceleration – acceleration due to interplay of quantum and EM theories.

Quantum vacuum state is providing naturally a Machian reference frame lost in classical limit. Critical acceleration in quantum theory (critical fields) leads to new particle production phenomena which have a good interpretation and no classical analog or obvious limit.

Quantum Vacuum Structure

The best known and widely accepted vacuum property is that it is a dielectric medium: a charge is screened by particle-hole (pair) excitations.



In Feynman language the real photon is decomposed into a bare photon and a photon turning into a virtual pair. The result is a renormalized electron charge smaller than bare, and slightly stronger Coulomb interaction (0.4% effect).

At any field strength the conversion of field energy into pairs possible.

Laws of Physics and Quantum Vacuum

Development of quantum physics leads to the recognition that vacuum fluctuations define laws of physics (Weinberg's effective theory picture). All this is nonperturbative property of the vacuum.

- The 'quantum æther' is polarizable: Coulomb law is modified; E.A.Uehling, 1935
- New interactions (anomalies) such as light-light scattering arise considering the electron, positron vacuum zero-point energy; Euler, Kockel, Heisenberg (1930-36);
- Casimir notices that the photon vacuum zero point energy also induces a new force, referred today as Casimir force 1949
- Non-fundamental vacuum symmetry breaking particles possible: Goldstone Bosons '60-s
- 'Fundamental electro-weak theory is effective - model of EW interactions, 'current' masses as VEV Weinberg-Salam '70-s
- Color confinement and high-T deconfinement Quark-Gluon Plasma '80-s

Some Time ago: QED of Strong Fields 1971–1979

The Decay of the Vacuum

Near a superheavy atomic nucleus empty space may become unstable, with the result that matter and antimatter can be created without any input of energy. The process might soon be observed experimentally

by Lewis P. Fulcher, Johann Rafelski and Abraham Klein

Scientific American 241, pp 150-9(1979), issue 6

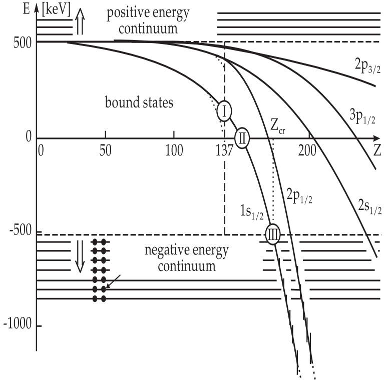
The vacuum is ordinarily defined as a state of absence: a vacuum is said to exist in a region of space if there is nothing in it. In the quantum field theories that describe the physics of elementary particles the vacuum becomes somewhat more complicated. Even in empty space matter can appear spontaneously as a result of fluctuations of the vacuum. For example, an electron and a positron, or antielectron, can be created out of the void. Particles created in this way have only a fleeting existence; they are annihilated almost as

in the space make the charge manifest. An electric field of sufficient intensity to create a charged vacuum is likely to be found in only one place: in the immediate vicinity of a superheavy atomic nucleus, one with about twice as many protons as the heaviest natural nuclei known. A nucleus that large cannot be stable, but it might be possible to assemble one for long enough to observe the decay of the neutral vacuum. Experiments that will test this possibility are now under way.

difference is carried away as energy by electromagnetic radiation.

The hydrogen binding energy of 13.6 electron volts is a small fraction of the rest mass of the electron, which is 511,000 electron volts, or about .5 MeV. (One MeV is a million electron volts.) The binding energy increases, however, along with the positive charge of the atomic nucleus. Such an increase is to be expected since a greater nuclear charge gives rise to a more intense electric field, and so the electron is bound more strongly. The nuclear charge is given by

Strong Fields in High Z Atoms



Single Particle Dirac Equation

$$(\vec{\alpha}\cdot i\vec{\nabla}+\beta m+V(r))\Psi_n(\vec{r}) = E_n\Psi_n(\vec{r})$$

$$V(r) = \begin{cases} -\frac{Z\alpha}{r} & r > R_N \\ -\frac{3}{2}\frac{Z\alpha}{R_N} + \frac{r^2}{2}\frac{Z\alpha}{R_N^3} & r < R_N \end{cases}$$

Key feature: bound states pulled from one continuum move as function of $Z\alpha$ across into the other continuum.

References: The large volume of work from 1968-85 is reviewed in W. Greiner, B. Müller and JR **“Quantum Electrodynamics of Strong Fields,”** (Springer Texts and Monographs in Physics, 1985), ISBN 3-540-13404-2.

QED of Strong Fields Book: 1986

W. Greiner B. Müller J. Rafelski

Quantum Electrodynamics of Strong Fields

With an Introduction into
Modern Relativistic Quantum Mechanics

With 258 Figures

Springer-Verlag
Berlin Heidelberg New York Tokyo

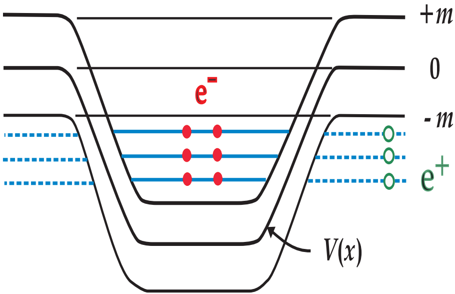
1. Introduction

The structure of the vacuum is one of the most important topics in modern theoretical physics. In the best understood field theory, Quantum Electrodynamics (QED), a transition from the neutral to a charged vacuum in the presence of strong external electromagnetic fields is predicted. This transition is signalled by the occurrence of spontaneous e^+e^- pair creation. The theoretical implications of this process as well as recent successful attempts to verify it experimentally using heavy ion collisions are discussed. A short account of the history of the vacuum concept is given. The role of the vacuum in various areas of physics, like gravitation theory and strong interaction physics is reviewed.

1.1 The Charged Vacuum

Our ability to calculate and predict the behaviour of charged particles in weak electromagnetic fields is primarily due to the relative smallness of the fine-structure constant $\alpha \approx 1/137$. However, physical situations exist in which the coupling constant becomes large, e.g. an atomic nucleus with Z protons can exercise a much stronger electromagnetic force on the surrounding electrons than could be described in perturbation theory, and hence it is foreseeable that the new expansion parameter $(Z\alpha)$ can quite easily be of the order of unity. In such cases non-perturbative methods have to be used to describe the resultant new phenomena, of which the most outstanding is the massive change of the ground-state structure, i.e. of the vacuum of quantum electrodynamics.

Formation of Charged Vacuum



Pair production across (nearly) constant field fills the 'dived' states available in the localized domain. 'Positrons' are emitted. Hence a localized charge density builds up in the vacuum reducing the field strength - back reaction.

Charged vacuum ground state observable by positron emission.

Rate W per unit time and volume of positron (pair) production in presence of a strong electric field $|\vec{E}|$ first made explicit by J. Schwinger, PRD82, 664 (1950).

$$W = \mathcal{I}m\mathcal{L}_{\rm eff} = \frac{c}{8\pi^3}\frac{(eE)^2}{(\hbar c)^4}\sum_{n=1}^{\infty}\frac{1}{n^2}e^{-\pi E_s/E}, \quad E_s = m_e^2c^3/e\hbar$$

What is special about E_s? For $E \to E_s$ vacuum unstable, pair production very rapid, field cannot be maintained.

Charged Vacuum: large strange quark matter $u - d - fs$ self neutralizes – very weekly charged: 1975-76

340 *J. Rafelski et al., Fermions and bosons interacting with arbitrarily strong external fields*

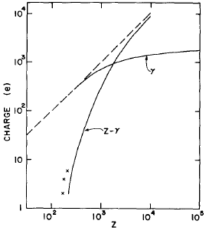


Fig. 6.2. The unscreened charge γ and the total charge of the vacuum $(Z - \gamma)$ as a function of Z. The crosses denote points from single particle calculations. The dashed line denotes the nuclear charge Z. From Müller and Rafelski [102].

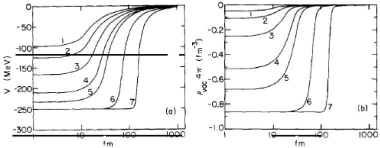


Fig. 6.3. The solutions of the relativistic Thomas–Fermi eq. (6.6) for selected values of the nuclear charge as a function of r. Curve 1, $Z = 600$: curve 2, 1000; curve 3, 2000; curve 4, 5000; curve 5, 10 000; curve 6, 10^5; curve 7, 10^6. (a) The self-consistent potential; (b) the corresponding charge distribution of the vacuum. From Müller and Rafelski [102].

Outlook: add Gravity + quantum world

$$\mathcal{J} = \mathcal{I}_{\sqrt{-g}} + \frac{1}{8\pi G}\int d^4x\sqrt{-g}\,R$$

There are acceleration paradoxes arising combining in this way gravity and electromagnetism:

- Charged electron in orbit around the Earth will not radiate if bound by gravitational field, but it will radiate had it been bend into orbit by magnets.
- A free falling electron near BH will not radiate but an electron resting on a surface of a table should (emissions outside observer's horizon).
- A micro-BH will evaporate, but a free falling observer may not see this: Is the BH still there? (Pisin Chen et al, PRL)

One is tempted to conclude that we do not have a theory incorporating acceleration. New Physics opportunity: find the theory – helped by experimental investigation of dynamics involving unit strength (critical) acceleration.

Acceleration and QVacuum Temperature

- W. Unruh finds that an accelerated observer records a temperature
$$T_{\rm HU} = \frac{a}{2\pi}$$
relation to Hawking radiation by the
strong acceleration $=$?? strong gravity connection
- We study properties of quantum vacuum 'accelerated' by a constant EM field – Since 1977 (B. Müller et al, PLA 63, p181) it is known that the effective Euler-Heisenberg action can be cast into a heat capacity format at temperature
$$T_{\rm EH} = \frac{a}{\pi}, \qquad a = eE/m$$
Resolution of the factor 2 difference with $T_{\rm HU}$ available: $g = 1$ particles. **L. Labun, JR, PRD86 (2012) 041701**

Summary

New opportunity to study foundational physics involving acceleration and search for generalization of laws of physics – motivated by need for better understanding of inertia, Mach's principle, Einstein's æther= the quantum vacuum .

Critical acceleration can be greatly exceeded in electron-laser pulse collisions.
Exploration of physical laws in a new domain possible

Experiments should help resolve the old riddle of EM theory and radiation reaction
Rich field of applications and theoretical insights follows

More generally, the study of physics phenomena beyond critical acceleration opens up to experimental exploration the frontier science addressing the consistency of General Relativity, Electromagnetism and Quantum Physics.