# Wujek Iwuniu
## “Versatile Scientist”

Jasiu Rafelski

THE UNIVERSITY OF ARIZONA

## “World Authority in QED and Quantum Optics”

Professor Iwo Bialynicki-Birula 90th birthday celebration

Organized by the Polish Academy of Science at the Faculty of Physics Warsaw University

Department of Energy
Washington, DC 20545

May 12, 1989

Professor Peter Carruthers, Chairman
Department of Physics
University of Arizona
Tucson, Arizona 85721

Dear Professor Carruthers:

This letter is to recommend Dr. Iwo Birula-Bialynicki for a position of a Visiting Professor at your Department.

I have known Dr. Birula for over thirty years and consider him Poland's foremost theoretical physicist. His leading position in Polish science has resulted in many awards and recognitions, including full membership in the Polish Academy of Sciences.

While most of Dr. Birula's published work is in the area of quantum electrodynamics, he is an extremely versatile scientist with excellent command of theoretical physics throughout the spectrum of this vast discipline. Dr. Birula's intimate grasp of physical phenomena results in his extraordinary ability to reduce complex physical systems to tractable and easily explainable models. He would add strength to any physics department, but his talent, expertise, and creativity make him an ideal match to the needs of Professor Rafelski's team.

I enthusiastically recommend Dr. Birula for a visiting professorship at the University of Arizona.

Sincerely,

Ryszard Gajewski

Ryszard Gajewski, Director
Division of Advanced Energy Projects
Office of Basic Energy Sciences, ER-16

5/18/89

University of Pittsburgh
FACULTY OF ARTS AND SCIENCES
Department of Physics and Astronomy

May 8, 1989

Dr. P. A. Carruthers, Head
University of Arizona
Department of Physics
Tucson, AZ 85721

Dear Peter:

I am pleased to support the appointment of Dr. Iwo Bialynicki-Birula to a position in your Department. Dr. Birula, as you know, is the head of the Institute for Theoretical Physics of the Academy of Sciences in Warsaw. He is a world authority in quantum electrodynamics and quantum optics. He has held a position as Adjunct Professor in the Department of Physics of the University of Pittsburgh for many years, and we have found his repeated visits to us extremely useful. These have also lead to joint cooperative programs, in which we have sent our faculty and students to Warsaw and benefitted from reciprocal visits in Pittsburgh. I am sure his appointment will be of similar use to the University of Arizona, and I strongly recommend it to you.

Best wishes,

Dick

R. H. Pratt
Professor of Physics

RHP:jat

# Thirty-five years ago: UA Colloquium April 27th 1988

**ARIZONA INN**

2200 East Elm Street
Tucson, Arizona 85719

Telephone
(602) 325-1541

BIRULA,PROF.I.B.
4614 5TH AVENUE
PITTSBURGH, PA.

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Iwo Białynicki ·
PhD 1959 Infeld/Warsaw
Postdoc 59-61 Rochester
Adj Prof. U of Pittsburg 72
Phys
Polish Academy of Sci
Royal Norweg. Acad. Sc.
Marie-Curie Sklodowsk

Twenty-nine years after his PhD with Prof. L. Infeld in 1959

A few anecdotes of how we met and became friends are worth telling.

And a few more anecdotes about my visit thirty-five years ago to Poland in late Fall of 1988.

On demand after lectures!

I. BIALYNICKI-BIRULA
University of Pittsburgh

Aug 11, 1988

Drogi Jasiu,

Rozmawiając z Rysiem, dosłownie w przeddzień mojego odlotu do Warszawy, dowiedziałem się, że zmieniłeś ostatnio zdanie (w porównaniu z tym co głosiłeś w czasie mojej wizyty w Tucson) i byłbyś skłonny przyjechać do Polski w tym roku.

Mam w związku z tym bardzo konkretną propozycję. Przyjedź na nasz Workshop w Centrum im. Stefana Banacha, który trwa od 19 września do 3 grudnia. Zdaję sobie sprawę, że tematyka tego Workshopu to jest to co Cię najbardziej pasjonuje, ale nie ma to chyba istotnego znaczenia (zaproszenie w załączeniu).

Spytaj przy okazji Petera Carruthersa czy się wybiera, bo za Twoją namową posłałem mu też zaproszenie. Nota bene: Twój dobry znajomy Berndt Müller obiecał przyjechać.

Kiedy ten list dotrze do Ciebie, ja będę już w Polsce. Gdybyś miał pytania przyślij telex albo zadzwoń (będę w Warszawie od 29 sierpnia, przed 29 sierpnia będę na wakacjach)

Nasz telex: 812468 ifpan

Telefon: instytut 470920 dom 430433

Adresy: Institut for Theoretical Physics, Lotników 32/46
02-668 Warszawa

PITTSBURGH, PA. 15260

dom Wernyhory 32, 02-727 Warszawa

Łączę serdeczne pozdrowienia Iwo

Gdy podejmiesz decyzję co do terminu przyjazdu, daj nam szybko znać. Najlepiej telexem, bo poczta jest powolna.

# 30-years ago: IBB-60: My DOE project to study vacuum structure using our DHW approach is not funded while work on quark-gluon plasma is. Grinding my teeth I moved away from vacuum-foundations to vacuum-applications.

| | | |
|---|---|---|
| Chemical freeze-out conditions in central S-S collisions at 200*A* GeV<br>J Sollfrank, M Gaździcki, U Heinz, J Rafelski<br>Zeitschrift für Physik C Particles and Fields 61, 659-665 | 177 | 1994 |
| Strangeness flow difference in nuclear collisions at 15A and 200A GeV<br>J Rafelski, M Danos<br>Physical Review C 50 (3), 1684 | 38 | 1994 |
| Formation and evolution of the quark-gluon plasma<br>J Letessier, J Rafelski, A Tounsi<br>Physics Letters B 333 (3-4), 484-493 | 30 | 1994 |
| Gluon production, cooling, and entropy in nuclear collisions<br>J Letessier, J Rafelski, A Tounsi<br>Physical Review C 50 (1), 406 | 92 | 1994 |
| Strangeness and particle freeze-out in nuclear collisions at 14.6 GeV A<br>J Letessier, J Rafelski, A Tounsi<br>Physics Letters B 328 (3), 499-505 | 74 | 1994 |
| Strange particle abundance in QGP formed in 200 GeV A nuclear collisions<br>J Letessier, J Rafelski, A Tounsi<br>Physics Letters B 323 (3-4), 393-400 | 46 | 1994 |
| Strange particle freeze-out<br>J Letessier, J Rafelski, A Tounsi<br>Physics Letters B 321 (4), 394-399 | 43 | 1994 |
| In Search of Entropy<br>J Rafelski, J Letessier, A Tounsi<br>Acta. Phys. Pol. A 85, 699 | 14 | 1994 |
| Hot hadronic matter: theory and experiment(Divonne, June 27- July 1, 1994)<br>J Letessier, H Gutbrod, J Rafelski<br>NATO Advanced Study Institute series. Series B, Physics | 5 | 1994 |
| Relativistic classical limit of quantum theory<br>GR Shin, J Rafelski<br>Physical Review A 48 (3), 1869 | 28 | 1993 |
| Evolution modes of the vacuum Wigner function in strong-field QED<br>I Białynicki-Birula, ED Davis, J Rafelski<br>Physics Letters B 311 (1-4), 329-338 | 17 | 1993 |
| Evidence for a phase with high specific entropy in nuclear collisions<br>J Letessier, A Tounsi, U Heinz, J Sollfrank, J Rafelski<br>Physical review letters 70 (23), 3530 | 137 | 1993 |


## Acta Physica Polonica A

przyrbwn.icm.edu.pl/APP/SPIS/a85-4.html

| Vol. 85 | No. 4 | April 1994 |
|---|---|---|

Proceedings of the International Symposium on Theoretical Physics
Dedicated to Iwo Białynicki-Birula in Honour of his 60th Birthday




### Introduction

This issue of Acta Physica Polonica is dedicated to Professor Iwo Białynicki-Birula, member of the Polish Academy of Sciences, on the occasion of his sixtieth birthday. The Centrum Fizyki Teoretycznej PAN organized an International Symposium on Theoretical Physics in Warsaw in honor of Professor Iwo Białynicki-Birula on 28 to 30 of October of 1993. With a few exceptions the papers published here contain the lectures delivered at the symposium. F. Persico has submitted his contribution but was unable to attend. J. Klauder spoke on a different subject.

Iwo Białynicki-Birula was born in Warsaw in 1933. He graduated from physics department of Warsaw University in 1956. Subsequently he received his Ph.D. there in 1959 under advice of Leopold Infeld. In 1962 he obtained the highest degree (habilitation) and soon became a Professor of Theoretical Physics in the Institute of Theoretical Physics of Warsaw University. He has held a number of visiting positions in the USA (Rochester, Pittsburgh, USC in Los Angeles, Tucson) and in Germany (Frankfurt). He is the author of over 200 research papers and several monographs on quantum electrodynamics and quantum mechanics. He is an elected member of Polish and Norwegian Academies of Sciences.

Iwo Białynicki-Birula is a very active theoretical physicist with particularly broad interests ranging from the field theory, general relativity and quantum electrodynamics, through quantum optics to plasma physics. He has always maintained intensive international scientific cooperative collaborations. He has particularly strong links with American physicists. The list of participants of the symposium reflects both his broad interests and contacts well.

We felt compelled to celebrate his anniversary for many reasons. Not only to express our appreciation for his role as one of the best Polish physicists. He is also an excellent educator and whole generations of Warsaw physicists enjoyed attending his demanding physics courses. After many years of teaching at the Warsaw University he has enthusiastically joined our new educational initiative at the Academy of Sciences: the College of Science launched in the fall of 1993.

Professor Białynicki-Birula is also a successful organizer. He proposed and organized our Centrum Fizyki Teoretycznej (former Institute for Theoretical Physics). He was also its first director setting us on the right track. Invaluable also was the role Białynicki-Birula played in helping us start our international carriers, learn the world and break the complexes of working in this remote country. This proved to be particularly valuable when, after the fall of communism, our country opened to the West. Suddenly all segments of our society are now expected to compete with the world. We are particularly well prepared for this competition. Hence you will find more optimism in the Centrum than almost anywhere else in the country. We appreciate Professor Iwo Białynicki-Birula's role in this.

I would like to acknowledge generosity of the KBN (National Committee for Scientific Research) which provided financial support for the symposium and this special issue of *Acta Physica Polonica A*. I thank also my colleagues from the Centrum for their help in the organization of the symposium. Finally, special thanks are due to Dr. Jerzy Kamiński of Warsaw University for his help in editorial work on this issue.

Kazimierz Rzążewski

**I found the November 8, 1991 lecture which was part of the attempted funding request to continue developing DHW vacuum structure, current comments in yellow background**

# From QED to Vlasov Equation

# RELATIVISTIC QUANTUM TRANSPORT THEORY

**A better title: DYNAMICS OF THE QED VACUUM IN EXTERNAL FIELDS**

*Phys. Rev. D44, 1825 (1991 - Sept. 15), and t.b.p.* **COLLABORATORS:**

- **Iwo Bialynicki-Birula and P. Gornicki, WARSAW**
- **E. David Davis and Ghi-Ryang Shin, Arizona**

**Our initial work: IBB, Pawel Gornicki, JR PRD44 1825(1991)** *Phase-space structure of the Dirac vacuum* **was followed by: Ghi Ryang Shin, IBB, JR PRA 46, 645 (1992)** *Wigner function of relativistic spin-1/2 particles*; **IBB, E.D. Davis, JR; PLB 311 (1993) 329** *Evolution modes of the vacuum Wigner function in strong field QED*; **Iwo returned to this topic: IBB, Łukasz Rudnicki PRD 83, 065020 (2011)** *Time evolution of the QED vacuum in a uniform electric field: Complete analytic solution by spinorial decomposition*; **followed by a short review (Wigner111 meeting in Budapest 2013): EPJ-W.Conf.78, 01001 (2014)** *Relativistic Wigner functions*; **continued in an unexpected manner: IBB, Zofia Bialynicki-Birula, PRA104, 022203 (2021)** *Time crystals made of electron-positron pairs*

**OBJECTIVE:**

**Relativistic Wigner Transform**

**WE SEEK TO DESCRIBE:**

**FLOW OF MATTER AND CONVERSION OF KINETIC ENERGY TO PARTICLE PAIRS IN A COMPLETE AND FUNDAMENTAL APPROACH**

$$\mathbf{W}(\vec{r},\vec{p},t) =? \int d^3s\, e^{-i\vec{p}\cdot\vec{s}/\hbar}\psi(\vec{r}+\vec{s}/2,t)\psi^*(\vec{r}-\vec{s}/2,t) \rightarrow$$

$$-\tfrac{1}{2}\int d^3s\, e^{-i\frac{\vec{p}\cdot\vec{s}}{\hbar}}\langle\Phi|e^{-i\frac{e}{\hbar}\int_{-1/2}^{1/2} d\lambda\, \vec{s}\cdot\vec{A}(\vec{r}+\lambda\vec{s},t)}[\Psi_\alpha(\vec{r}+\vec{s}/2,t),\Psi^\dagger_\beta(\vec{r}-\vec{s}/2,t)]|\Phi\rangle$$

**NOTE: Symmetric $\{\psi,\psi^\dagger\}$ 'used up' in quantization**

**The novel element is the appearance of gauge invariance securing phase inspired by work of Ken Johnson in 60's**

## $4 \times 4 = 16$ WIGNER FUNCTION COMPONENTS:

$$\mathbf{W}(\vec{r},\vec{p},t) = \frac{1}{4}[f_0 + \sum_{i=1}^{3} \rho_i f_i + \vec{\sigma}\cdot\vec{g}_0 + \sum_{i=1}^{3} \rho_i \vec{\sigma}\cdot\vec{g}_i]$$

$$\begin{array}{cccccccc}
f_0, & f_1, & f_2, & f_3, & \vec{g}_0, & \vec{g}_1, & \vec{g}_2, & \vec{g}_3 \\
\psi^\dagger(1, & \rho_1, & \rho_2, & \rho_3, & \sigma_k, & \rho_1\sigma_k, & \rho_2\sigma_k, & \varepsilon^{ijk}\rho_3\sigma_k)\psi \\
\bar{\psi}(\gamma^0, & i\gamma^0\gamma_5, & \gamma_5, & 1, & -i\gamma_5\gamma^k, & \gamma^k, & -i\gamma^0\gamma^k, & i\,\gamma^{ij})\psi
\end{array}$$

- $\bar{\psi}\gamma_\mu\psi \rightarrow (f_0, \vec{g}_1)$;  $\bar{\psi}\psi \rightarrow f_3$
- $\bar{\psi}\gamma_5\gamma^\mu\psi \rightarrow (f_1, \vec{g}_0)$;  $\bar{\psi}\gamma_5\psi \rightarrow f_2$
- $\vec{g}_2, \vec{g}_3$ **- electric & magnetic moment density**

**30y and counting: Our approach remains unique characterization of phase space allowing for spin and particles/antiparticles: one → 16 dynamic phase space distribution components**

**WE KEEP MATTER FIELD FLUCTUATIONS**

**'STRONG E-M FIELD APPROXIMATION'**

$$W_{\alpha\beta}(\vec{r},\vec{p},t) = -\tfrac{1}{2}\int d^3s e^{-i\vec{p}\cdot\vec{s}/\hbar} e^{-i\frac{e}{\hbar}\int_{-1/2}^{1/2} d\lambda\, \vec{s}\cdot\vec{A}(\vec{r}+\lambda\vec{s},t)} \langle\Phi|[\Psi_\alpha(\vec{r}_+,t), \Psi_\beta^\dagger(\vec{r}_-,t)]|\Phi\rangle$$

**relation to FEYNMAN PROPAGATOR: IF** $|0^{in}\rangle = |0^{out}\rangle = |\Phi\rangle \rightarrow$
$W_{\alpha\beta}(\vec{r},\vec{p},t) = \frac{i}{2}\int d^3s\, \exp(-i\vec{p}\cdot\vec{s}/\hbar) G_{\mathrm{F}\,\alpha\beta}(\vec{r}+\vec{s}/2,t,\vec{r}-\vec{s}/2,t)\gamma^0$ **with**

$$-iG_{\mathrm{F}}(\vec{r},t;\vec{r}\,',t') = \langle 0^{out}|\exp(i\frac{e}{\hbar}\int_{\vec{r},t}^{\vec{r}\,',t'} d\xi^\mu A_\mu(\xi))\, T(\psi(\vec{r},t)\,\bar{\psi}(\vec{r}\,',t')|0^{in}\rangle$$

**WIGNER FUNCTION satisfies** $(\vec{\alpha} = \rho_1\vec{\sigma}, \beta = \rho_3)$**:**

$$i\hbar\mathbf{D}_t\mathbf{W} = -i\hbar c\vec{\mathbf{D}}\cdot\frac{1}{2}\{\vec{\alpha},\mathbf{W}\} + c[\vec{\alpha}\cdot\vec{\mathbf{P}} + \beta mc, \mathbf{W}]$$

**My contribution to this Eq. was a vehement rejection of 1+1d version 'nobody will care about'. Iwo took this to his heart and derived 1+3 d overnight; Classical Wigner function limit needs '$i$' and '$\hbar$' always visible**

$$
\begin{aligned}
\mathbf{D}_t &= \partial_t + e\int_{-1/2}^{1/2} d\lambda \vec{E}(\vec{r} + i\hbar\lambda\vec{\partial}_p, t)\cdot\vec{\partial}_p \\
\vec{\mathbf{D}} &= \vec{\nabla} + e\int_{-1/2}^{1/2} d\lambda \vec{B}(\vec{r} + i\hbar\lambda\vec{\partial}_p, t)\times\vec{\partial}_p \\
\vec{\mathbf{P}} &= \vec{p} - ie\hbar\int_{-1/2}^{1/2} d\lambda\,\lambda\,\vec{B}(\vec{r} + i\hbar\lambda\vec{\partial}_p, t)\times\vec{\partial}_p
\end{aligned}
$$

**THE CLASSICAL LIMIT $\hbar \to 0$**

$$
\begin{aligned}
\mathbf{D}_t &= \partial_t + e\vec{E}(\vec{r},t)\cdot\vec{\partial}_p - \frac{e\hbar^2}{12}(\vec{\nabla}\cdot\vec{\partial}_p)^2\vec{E}(\vec{r},t)\cdot\vec{\partial}_p + \ldots \\
\vec{\mathbf{D}} &= \vec{\nabla} + e\vec{B}(\vec{r},t)\times\vec{\partial}_p - \frac{e\hbar^2}{12}(\vec{\nabla}\cdot\vec{\partial}_p)^2\vec{B}(\vec{r},t)\times\vec{\partial}_p + \ldots \\
\vec{\mathbf{P}} &= \vec{p} + \frac{e\hbar^2}{12}(\vec{\nabla}\cdot\vec{\partial}_p)\,\vec{B}(\vec{r},t)\times\vec{\partial}_p + \ldots
\end{aligned}
$$

**ALL OPERATORS ARE REAL VALUED**

# COMPONENTS of W SATISFY:

$$
\begin{aligned}
\mathbf{D}_t f_0 + c\vec{\mathbf{D}}\cdot\vec{g}_1 &= 0\\
\mathbf{D}_t f_1 + c\vec{\mathbf{D}}\cdot\vec{g}_0 &= -2\frac{mc^2}{\hbar}f_2\\
\mathbf{D}_t f_2 + 2\frac{c}{\hbar}\vec{\mathbf{P}}\cdot\vec{g}_3 &= 2\frac{mc^2}{\hbar}f_1\\
\mathbf{D}_t f_3 - 2\frac{c}{\hbar}\vec{\mathbf{P}}\cdot\vec{g}_2 &= 0\\
\mathbf{D}_t\vec{g}_0 + \vec{\mathbf{D}}f_1 - 2\frac{c}{\hbar}\vec{\mathbf{P}}\times\vec{g}_1 &= 0\\
\mathbf{D}_t\vec{g}_1 + c\vec{\mathbf{D}}f_0 - 2\frac{c}{\hbar}\vec{\mathbf{P}}\times\vec{g}_0 &= -2\frac{mc^2}{\hbar}\vec{g}_2\\
\mathbf{D}_t\vec{g}_2 + c\vec{\mathbf{D}}\times\vec{g}_3 + 2\frac{c}{\hbar}\vec{\mathbf{P}}f_3 &= 2\frac{mc^2}{\hbar}\vec{g}_1\\
\mathbf{D}_t\vec{g}_3 - \vec{\mathbf{D}}\times\vec{g}_2 - 2\frac{c}{\hbar}\vec{\mathbf{P}}f_2 &= 0
\end{aligned}
$$

$f_0 \leftrightarrow \rho,\ \vec{g}_1 \leftrightarrow \vec{j},\ f_3 \leftrightarrow s,\ f_1 \leftrightarrow \rho_{\mathcal{P}},\ \vec{g}_0 \leftrightarrow \vec{j}_{\mathcal{P}},\ f_2 \leftrightarrow s_{\mathcal{P}},\ \vec{g}_3 \leftrightarrow \vec{\mu},\ \vec{g}_2 \leftrightarrow \vec{\mu}_{\mathcal{E}}$

**REAL VALUED PHASE SPACE DISTRIBUTIONS**

**MAXWELL:**

$$
\begin{aligned}
\partial_t \vec{B} &= -\vec{\nabla} \times \vec{E} \\
\vec{\nabla} \cdot \vec{B} &= 0 \\
\partial_t \vec{D} &= \vec{\nabla} \times \vec{H} - \vec{j} \\
\vec{\nabla} \cdot \vec{D} &= \rho \\
\rho(\vec{r}, t) &= e \int d\mathbf{p}\, f_0(\vec{r}, \vec{p}, t) + \rho_{ext}(\vec{r}, t) \\
\vec{j}(\vec{r}, t) &= e \int d\mathbf{p}\, \vec{g}_1(\vec{r}, \vec{p}, t) + \vec{j}_{ext}(\vec{r}, t)
\end{aligned}
$$

$d\mathbf{p} = \frac{d^3p}{(2\pi\hbar)^3}$, $\vec{D} = \epsilon_0 \vec{E}$ , $\vec{H} = \mu_0^{-1} \vec{B}$

**However:** $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$, $\vec{H} = \mu_0^{-1} \vec{B} - \vec{M}$: **We did not INCLUDE induced polarization** $\vec{P}$ **and magnetization** $\vec{M}$ **except when we dealt (see below) with renormalization; in lowest order in linear response for constant fields:**

$$
\begin{aligned}
\vec{P} &= e \int d\mathbf{p}\, \vec{g}_2(\vec{r}, \vec{p}, t) \qquad \vec{g}_2 = \frac{em\vec{E}}{E_p^3} \\
\vec{M} &= e \int d\mathbf{p}\, \vec{g}_3(\vec{r}, \vec{p}, t) \qquad \vec{g}_3 = -\frac{em\vec{B}}{E_p^3}
\end{aligned}
$$

**which terms are to be 'subtracted'.**

## SIMPLE SOLUTIONS & SOME GENERAL PHYSICAL PROPERTIES

- **FREE VACUUM: all space derivatives and fields $\vec{E}$, $\vec{B}$ vanish**

$$0 = [\vec{\alpha}\cdot\vec{p} + \beta m, \mathbf{W}^0] \to \mathbf{W}^0 = -\frac{\vec{\alpha}\cdot\vec{p} + \beta m}{2E_p}$$

$E_p = \sqrt{m^2 + \vec{p}^{\,2}}$; **normalization** $1/2E_p$ **from definition of W**

$$f_3^0 = -\frac{2m}{E_p}\;; \qquad \vec{g}_1^{\,0} = \frac{\vec{p}}{m} f_3^0$$

**The dynamical equations for homogenous vacuum state imply:** $\vec{p}f_3 = m\vec{g}_1$, $\vec{p}\times\vec{g}_1 = 0$ $\to$ **solution seen above also following directly from taking** $W \propto \vec{\alpha}\cdot p + \beta m$. **HOWEVER what about other functions: no dynamical constraint for** $f_0$ **so we can CHOOSE vacuum state to have zero charge (usual choice). We further see** $f_2 = 0$, **so pseudoscalar density must be zero. HOWEVER: 1)** $\vec{p}\cdot\vec{g}_3 = mcf_1$, **nothing forces pseudovector charge density** $f_1$ **to vanish if it is accompanied by spontaneous** $g_3$ **(magnetization). 2) We have** $\vec{p}\times\vec{g}_0 = mc\vec{g}_2$, $\vec{p}\cdot\vec{g}_2 = 0$ **which implies that IF there is in vacuum spontaneous symmetry breaking pseudovector current** $\vec{g}_0$, **the electric polarizability** $g_2$ **will not vanish. CONCLUSION: we chose a vacuum with scalar density and vector current, omitting two other combinations which are permissible but involve spontaneous parity breaking, a feature which should be explored.**

- **CLASSICAL LIMIT $\hbar = 0$: VLASOV Eq.**
  **ANSATZ: Pseudo-scalar&vector, elec&magn polarization = 0; remain:**
  1. $\mathbf{D}_t f_0 + c\vec{\mathbf{D}} \cdot \vec{g}_1 = 0$
  2. $\mathbf{D}_t \vec{g}_2 + c\vec{\mathbf{D}} \times \vec{g}_3 + 2\frac{c}{\hbar}\vec{\mathbf{P}} f_3 = 2\frac{mc^2}{\hbar}\vec{g}_1$
  3. **use the classical relation:** $f_3 = m/E_p f_0$: $f_0 \equiv f;\ f_3 \to \frac{m}{E_p} f$;
  4. $\mathbf{D}_t f + \vec{\mathbf{D}} \cdot \vec{p}(f/E_p) = 0$ **with** $\hbar = 0$

  **THIS IS THE REL. VLASOV EQUATION OF PLASMA THEORY:**

$$\partial_t f + \vec{v} \cdot \vec{\nabla} f + e(\vec{E} + \vec{v} \times \vec{B}) \cdot \vec{\partial}_p f = 0,$$

  **with relativistic** $\boxed{\vec{v} = \vec{p}/E_p}$. **FACET:**
  **Vlasov is the next level approximation after free field case.**
  **DESCRIBES ONLY FLOW OF MATTER ('acoustical' mode)**
  **NO PARTICLE PRODUCTION.**

- **AXIAL ANOMALY**

**Iwo, why did we never publish this? A popular chiral effect driven by external fields**

**chiral (4-vector) current** $j_5^\mu = \overline{\psi}\gamma^\mu\gamma_5\psi \to -(f_1,\, \vec{g}_0)$
**chiral density** $j_5 = \overline{\psi}\gamma_5\psi$ $(\gamma_5 = \gamma^0\gamma^1\gamma^2\gamma^3) \to f_2$

$$\begin{aligned} -\partial_t f_1 - \vec{\nabla}\cdot\vec{g}_0 - 2mf_2 \;&=\; e\int_{-1/2}^{1/2} d\lambda\, \vec{E}(\vec{r}_\lambda, t)\cdot\vec{\partial}_p\, f_1 \\ &+\; e\int_{-1/2}^{1/2} d\lambda \left(\vec{B}(\vec{r}_\lambda, t)\times\vec{\partial}_p\right)\cdot\vec{g}_0 \end{aligned}$$

**LHS should naively vanish, phase space equivalent of** $\partial^\mu j_\mu^5 - 2mj^5 \to$ **anomaly in QED. BUT: take lowest order** $f_1^{(1)}, \vec{g}_0^{(1)}$**:**

$$-\partial_t f_1 \;-\; \vec{\nabla}\cdot\vec{g}_0 \;-\; 2mf_2 \;=\; -\frac{3}{E_p^5}\, m^2 e^2\, \vec{E}\cdot\vec{B} \;+\; \ldots$$

$f_0 \leftrightarrow \rho,\ \vec{g}_1 \leftrightarrow \vec{j},\ f_1 \leftrightarrow \rho_{\mathcal{P}},\ \vec{g}_0 \leftrightarrow \vec{j}_{\mathcal{P}},\ f_2 \leftrightarrow s_{\mathcal{P}}$ **; All parity breaking functions**

- **CONSERVATION LAWS:**$(d\Gamma = d\mathbf{r}\, d\mathbf{p} = (2\pi\hbar)^{-3}\, d^3r\, d^3p)$

$$\textbf{Total charge } \partial_t Q = 0 \qquad Q = e\int d\Gamma\, f_0(\vec r, \vec p, t)$$

$$\textbf{total energy } \partial_t E = 0 \qquad \textbf{momentum } \partial_t \vec P = 0$$

$$E = \int d\Gamma[\vec p \cdot \vec g_1 + m f_3] \;+\; \tfrac{1}{2}\int d\mathbf{r}\,[\vec E\cdot\vec D + \vec B\cdot\vec H]$$

$$\vec P = \int d\Gamma\, \vec p f_0 \;+\; \int d\mathbf{r}\,[\vec D \times \vec B]$$

$$\textbf{angular momentum } \partial_t \vec M = 0 \qquad \textbf{boost generator } \partial_t \vec N = 0$$

$$\vec M = \int d\Gamma[\vec r \times \vec p f_0 + \tfrac{\hbar}{2}\vec g_0] \;+\; \int d\mathbf{r}\, \vec r \times [\vec D \times \vec B]$$

$$\vec N = \int d\Gamma\, \vec r\,[\vec p \cdot \vec g_1 + m f_3] \;+\; \tfrac{1}{2}\int d\mathbf{r}\, \vec r\,[\vec E\cdot\vec D + \vec B\cdot\vec H] - t\,\vec P$$

**ALSO CONSERVED:** $\partial_t \int d\Gamma(\sum f_i^2 + \sum g_i^2) = 0$

**The rôle of the last conservation law not understood. Polarization entering the $\vec D$ field and magnetization in $\vec H$ field were not looked at.**

- **HOMOGENEOUS MAGNETIC FIELD:**

$$(f_1,\ f_3,\ \vec{g}_1,\ \vec{g}_3) = -\frac{2}{\sqrt{\pi}}\int_{-\infty}^{\infty} d\kappa\ \exp\left(-\kappa^2 m_\parallel^2 - \tanh(\kappa^2\mathcal{B})p_\perp^2/\mathcal{B}\right) \times$$
$$\left(p_\parallel \tanh(\kappa^2\mathcal{B}),\quad m,\quad \vec{p}-\vec{p}_\perp \tanh^2(\kappa^2\mathcal{B}),\quad m\tanh(\kappa^2\mathcal{B})\vec{B}/|B|\right)$$

**where:** $\mathcal{B} = |e\vec{B}|,\ \ m_\parallel = \sqrt{m^2 + p_\parallel^2}$**; for small fields:** $(f_1,\ f_3,\ \vec{g}_1,\ \vec{g}_3) \approx$

$$-\left(e\frac{\vec{p}\cdot\vec{B}}{E_p^3},\quad \frac{2m}{E_p} + \frac{5m(eB)^2 p_\perp^2}{4E_p^7},\quad \frac{2\vec{p}}{E_p} + \frac{5\vec{p}\,(eB)^2 p_\perp^2}{4E_p^7} - \frac{3\vec{p}_\perp(eB)^2}{2E_p^5},\quad \frac{m\,e\vec{B}}{E_p^3}\right)$$

**We did not look 'beyond' at this most interesting solution! Vacuum in ultrastrong beyond 'critical' magnetic fields present in magnetars**

- **ELECTRIC FIELD - PAIR PRODUCTION**

**In contrast to Vlasov flow we seek particle generation. Set $\vec{B}=0$:**

$$i\hbar \mathbf{D}_t \mathbf{W} = -i\hbar \vec{\nabla}\{\vec{\alpha}, \mathbf{W}\} + [\vec{\alpha}\cdot\vec{p} + \beta m, \mathbf{W}]$$

**CONSTANT ELECTRIC FIELD**

$$\mathbf{W}(\vec{r},\vec{p},t) \to \mathbf{W}^E(\vec{p},t)\ , \qquad \mathbf{D}_t = \partial + e\vec{E}\cdot\vec{\partial}_p \to$$

$$i\hbar(\partial_t + e\vec{E}\cdot\vec{\partial}_p)\mathbf{W}^E = [\rho_1\vec{\sigma}\cdot\vec{p} + \rho_3 m, \mathbf{W}^E] \qquad \textbf{hence}$$

$$\mathbf{W}^E = \frac{1}{4}[\rho_3 f_3 + \vec{\sigma}\cdot\vec{g}_0 + \rho_1\vec{\sigma}\cdot\vec{g}_1 + \rho_2\vec{\sigma}\cdot\vec{g}_2]$$

**$f_0=0$ no charge density, $f_1=0$ no pseudo charge density, $f_2=0$ no pseudoscalar density, $\vec{g}_3=0$ no mag. mom. density.**

**With $\vec{E}\neq 0$ vacuum structure REQUIRES beyond vacuum $f_3$ (scalar) and $\vec{g}_1$ (flow 3vector) furthermore the $\vec{g}_0$ (pseudo3vector) and $\vec{g}_2$ (electric polarizability). As noted earlier dynamical equation is $\vec{p}\cdot\vec{g}_3 = mcf_1$, nothing forces pseudovector charge density $f_1$ to vanish if it is accompanied by (spontaneous) $g_3$ (magnetization).**

**Consider the 10-dimensional vector:** $W_{10} = (f_3, \vec{g}_0, \vec{g}_1, \vec{g}_2)^{\mathbf{T}}$

$$(\partial_t + e\vec{E} \cdot \vec{\partial}_p)\, W_{10} + M(\vec{p})\, W_{10} = 0,$$

$$M(\vec{p}) = \begin{pmatrix} 0 & 0 & 0 & -2\vec{p} \\ 0 & 0 & -2\times\vec{p} & 0 \\ 0 & -2\times\vec{p} & 0 & 2m \\ 2\vec{p} & 0 & -2m & 0 \end{pmatrix}$$

**solve numerically: classical evolution** $\frac{d\vec{p}(t)}{dt} = e\vec{E}(t)$

**Initial condition:** $\vec{p}(\vec{p}_0|t=0) = \vec{p}_0 \to \vec{p}(\vec{p}_0|t) = \vec{p}_0 + e\int dt\, \vec{E}(t)$

**We obtained Euler-Heisenberg-Schwinger pair production with some help from Feynman. This was much more elegantly addressed by: IBB, Łukasz Rudnicki, PRD 83, 065020 (2011)** *Time evolution of the QED vacuum in a uniform electric field: Complete analytic solution by spinorial decomposition* **I stop therefore after this one pertinent slide from 1991 lecture**

- **RENERMALIZATION: Components of the DHW function do not fall-off sufficiently fast for large momenta $\rightarrow$ charge renormalization:**

**We need large-$p$ 'perturbative' subtraction terms to assure that all phase space integrals yield finite spatial densities.**

## PERTURBATIVE SOLUTION

$$f_0 = -\frac{e}{2E_p^3}\left(\vec{\nabla}\cdot\vec{E} - \frac{(\vec{p}\cdot\vec{\nabla})(\vec{p}\cdot\vec{E})}{E_p^2}\right), \qquad f_1 = -\frac{e}{E_p^3}\,\vec{p}\cdot\vec{B}, \qquad f_2 = 0,$$

$$f_3 = -\frac{2m}{E_p} + \frac{em}{2E_p^5}\,\vec{p}\cdot(\vec{\nabla}\times\vec{B}), \qquad \vec{g}_0 = \frac{e}{E_p^3}\,\vec{p}\times\vec{E},$$

$$\vec{g}_1 = -\frac{2\vec{p}}{E_p} - \frac{e}{2E_p^3}\left(\frac{4}{3}\vec{\nabla}\times\vec{B} - \frac{(\vec{p}\cdot\vec{\nabla})(\vec{p}\times\vec{B})}{E_p^2} - \frac{\vec{p}\ \vec{p}\cdot(\vec{\nabla}\times\vec{B})}{E_p^2}\right),$$

$$\vec{g}_2 = \frac{em\vec{E}}{E_p^3}, \qquad \vec{g}_3 = -\frac{em\vec{B}}{E_p^3}.$$

**We obtained above all 16 phase space functions with leading terms in the field, naively only $f_0$ and $\vec{g}_1$ contribute induced charge density and 3-current. Terms in $1/E_p^3$ 'OK'= logarithmically divergent, more singular terms 'rescued' by angular integrals, or are part of 'free' vacuum.**

## Polarization charge, current: $\rho_{pol}, \vec{j}_{pol}$:

$$
\begin{aligned}
\rho_{pol} &= e\int d\mathbf{p}\, f_0(\vec{r},\vec{p},t) \simeq -\frac{e^2}{4\pi^2}\int_0^\Lambda dp\, \frac{p^2}{E_p^3}\left(1-\frac{p^2}{3E_p^2}\right)\ \vec{\nabla}\cdot\vec{E} \\
\vec{j}_{pol} &= e\int d\mathbf{p}\, \vec{g}_1(\vec{r},\vec{p},t) \simeq -\frac{e^2}{4\pi^2}\int_0^\Lambda dp\, \frac{p^2}{E_p^3}\left(\frac{4}{3}-\frac{2p^2}{3E_p^2}\right)\ \vec{\nabla}\times\vec{B}
\end{aligned}
$$

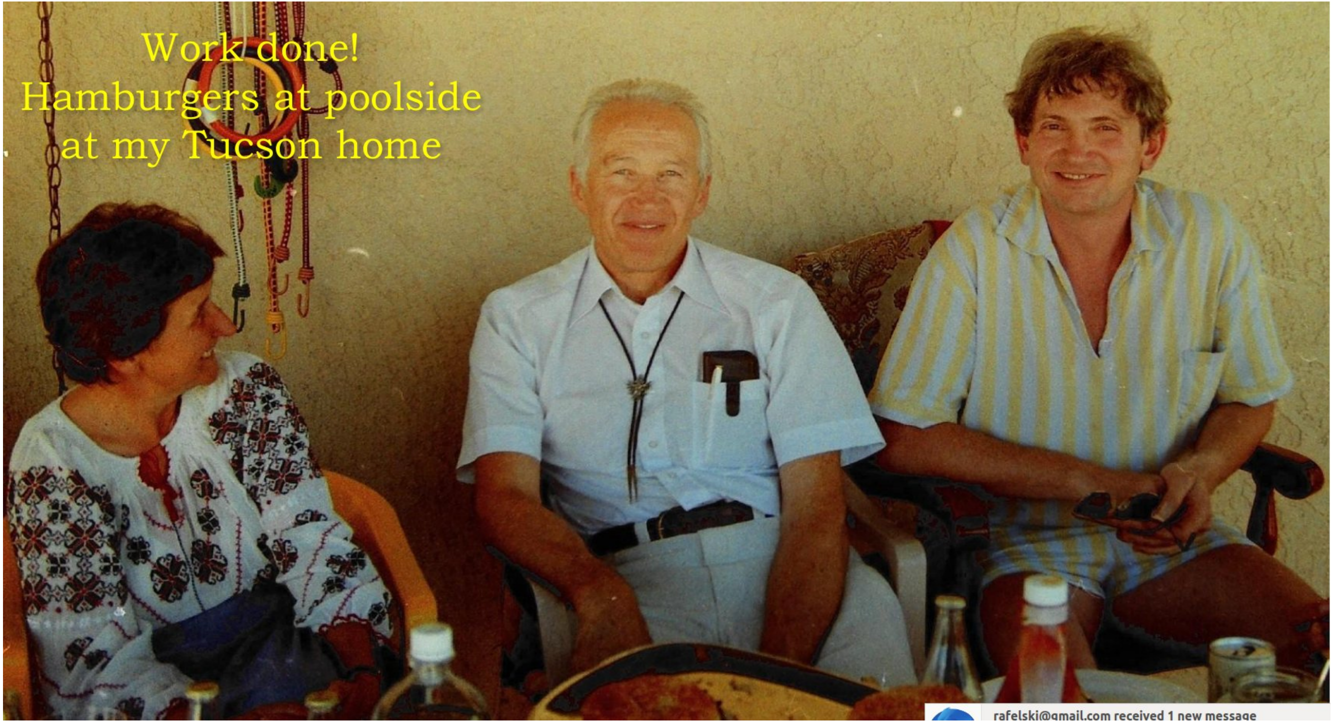
## Logarithmic divergence: We needed momentum cut-off $\Lambda$;

$$
\begin{aligned}
\vec{\nabla}\cdot\vec{E}\left(\epsilon_0+\frac{e^2}{24\pi^2}\ln(\Lambda/m)\right) &= \rho_{ext}, \\
\vec{\nabla}\times\vec{B}\left(\mu_0^{-1}+\frac{e^2}{24\pi^2}\ln(\Lambda/m)\right) &= \vec{j}_{ext}.
\end{aligned}
$$

**The terms dependent on $\Lambda$ absorbed by redefining the permittivity $\epsilon_0$ and the permeability of the vacuum $\mu_0$:**

$$\epsilon_v = \epsilon_0 \left(1 + \frac{\alpha}{6\pi} \ln(\Lambda/m)\right),$$
$$\mu_v = \mu_0 \left(1 + \frac{\alpha}{6\pi} \ln(\Lambda/m)\right)^{-1}.$$
$$\alpha_{phys} = \frac{e^2}{4\pi\hbar c\epsilon_v}$$
$$c_{phys} = c_0$$

**Recall mention of polarization functions $\vec{g}_2$ and $\vec{g}_3$ which directly renormalize $\epsilon_0$ and $\mu_0^{-1}$, no detour via charge and current - so this situation needs more thought, also since there are other logarithmically divergent vacuum structure functions. This suggests a thorough 2nd look is needed to understand the meaning of all vacuum perturbative terms.**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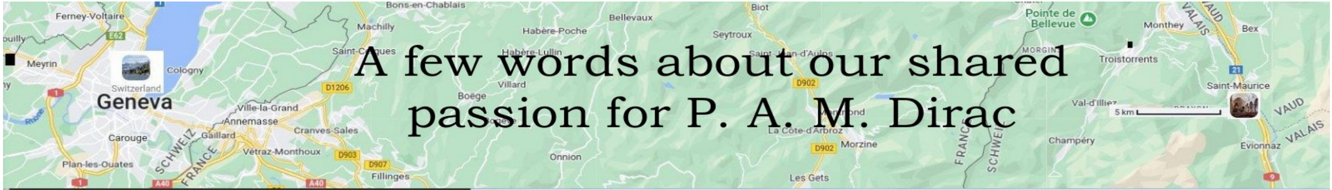
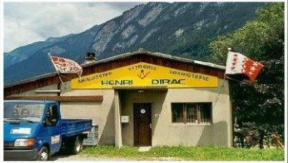
# A few words about our shared passion for P. A. M. Dirac




## Paul Dirac's garden

This garden is in honor of Paul Dirac (1902-1984), a native of Saint-Maurice. He received the Nobel Prize in Physics in 1933. Paul Dirac was born in Great Britain of a Swiss father naturalized in England and an English mother. He taught and carried out his research in England and then in Florida.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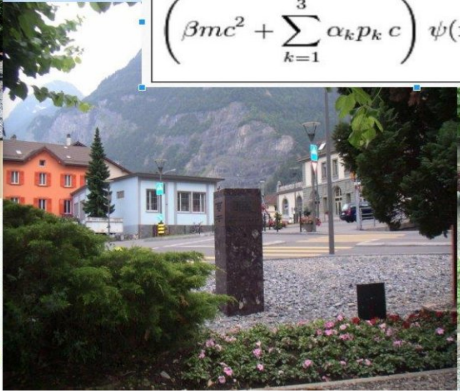
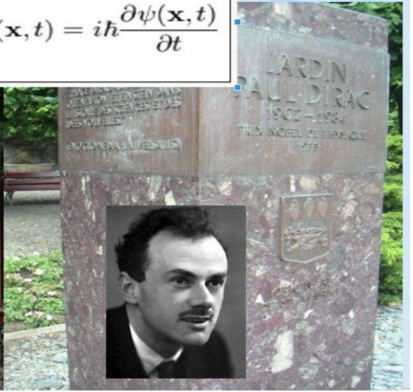
## Relativity enters the quantum world: Paul Dirac -Family roots in St Maurice, VS

The Dirac equation in the form originally proposed is

$$\left(\beta mc^2 + \sum_{k=1}^{3} \alpha_k p_k \, c\right) \psi(\mathbf{x}, t) = i\hbar \frac{\partial \psi(\mathbf{x}, t)}{\partial t}$$





# Our common passion for vacuum structure

**Johann Rafelski, Berndt Muller**

The Structured Vacuum: Thinking About Nothing

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Phase-space structure of the Dirac vacuum

I Bialynicki-Birula, P Gornicki, J Rafelski - Physical Review D, 1991 - APS

We study the phase-space correlation function for the Dirac vacuum in the presence of simple field configurations. Our formalism rests on the Wigner transform of the Dirac-Heisenberg correlation function of the Dirac field coupled to the electromagnetic field. A self-consistent set of equations obeyed by the 16 components of the phase-space correlation function and by the electric and magnetic field is derived. Our approach is manifestly gauge invariant. A closed system of integro-differential equations is obtained neglecting the …

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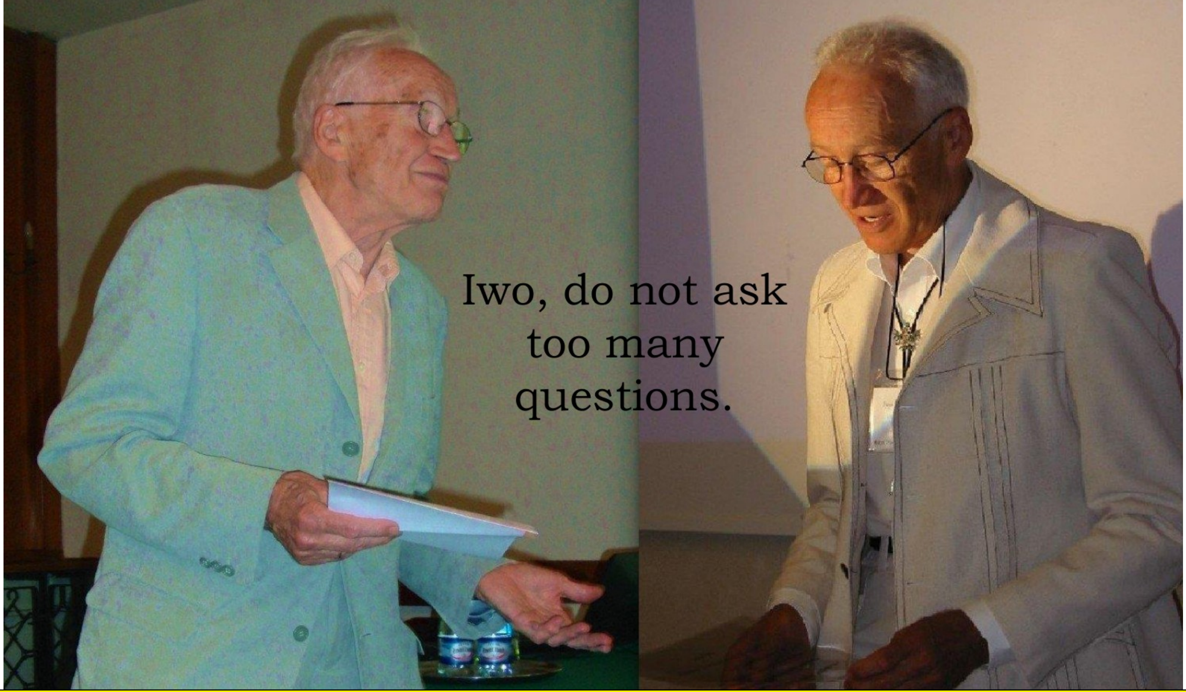


**Instead: We write this week the overdue DHW BR-Paper-3**