

Quarks to Cosmos: Particles and Plasma in Cosmological Evolution

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Budapest; Photo by László Pál Csernai 2019



Purpose and colleagues

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We want to use the knowledge of particle and nuclear physics gained in the last 50 years to describe the Universe's evolution after electroweak symmetry breaking at ~ 130 GeV. We believe that without a detailed understanding of the Known, the vast volume of work on the Unknown content of the Universe are literal random shots in the darkness.



Prof. Johann Rafelski



Prof. Andrew Steinmetz



Prof. Jeremiah Birrell



Dr. Christopher Grayson

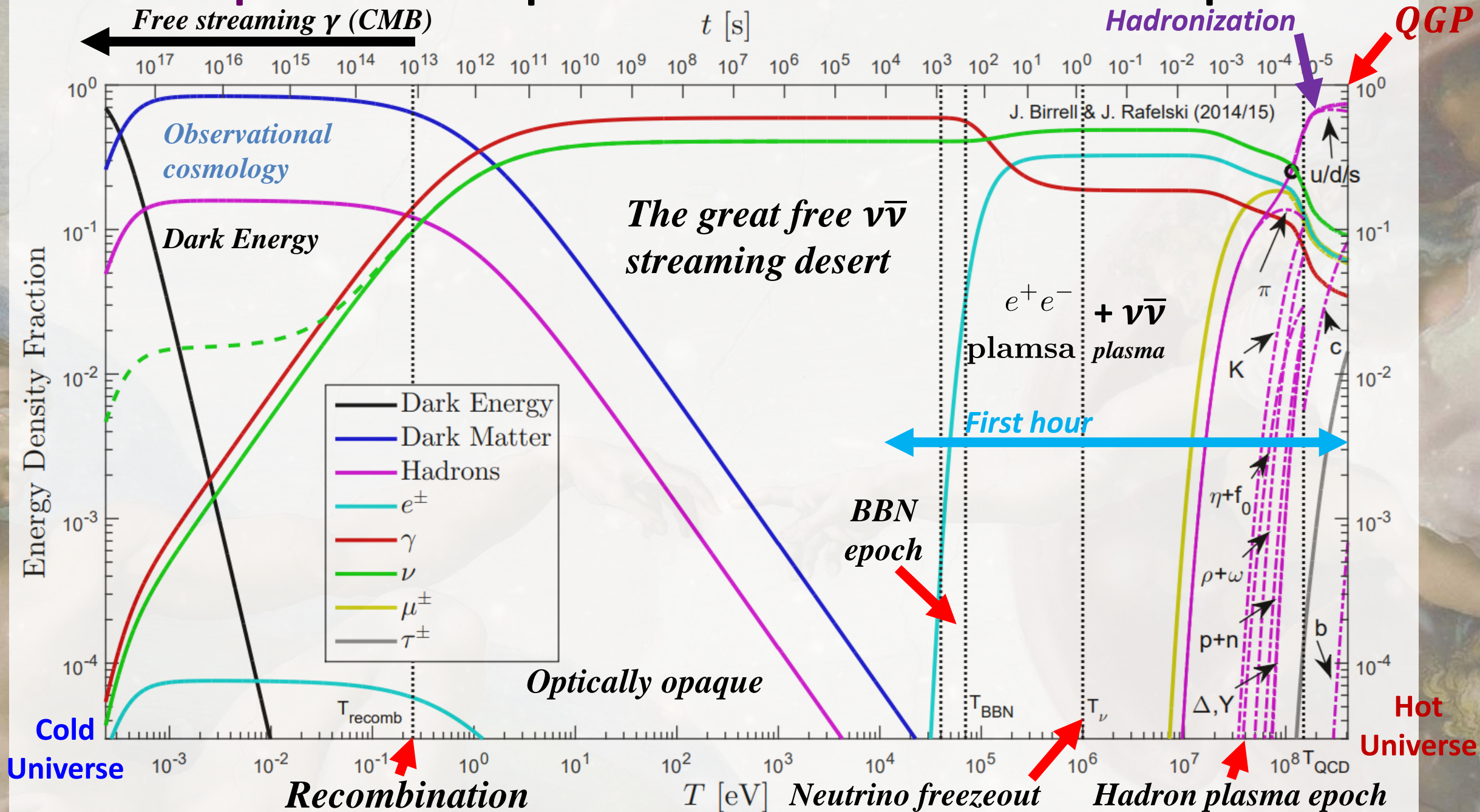


Dr. Cheng Tao Yang

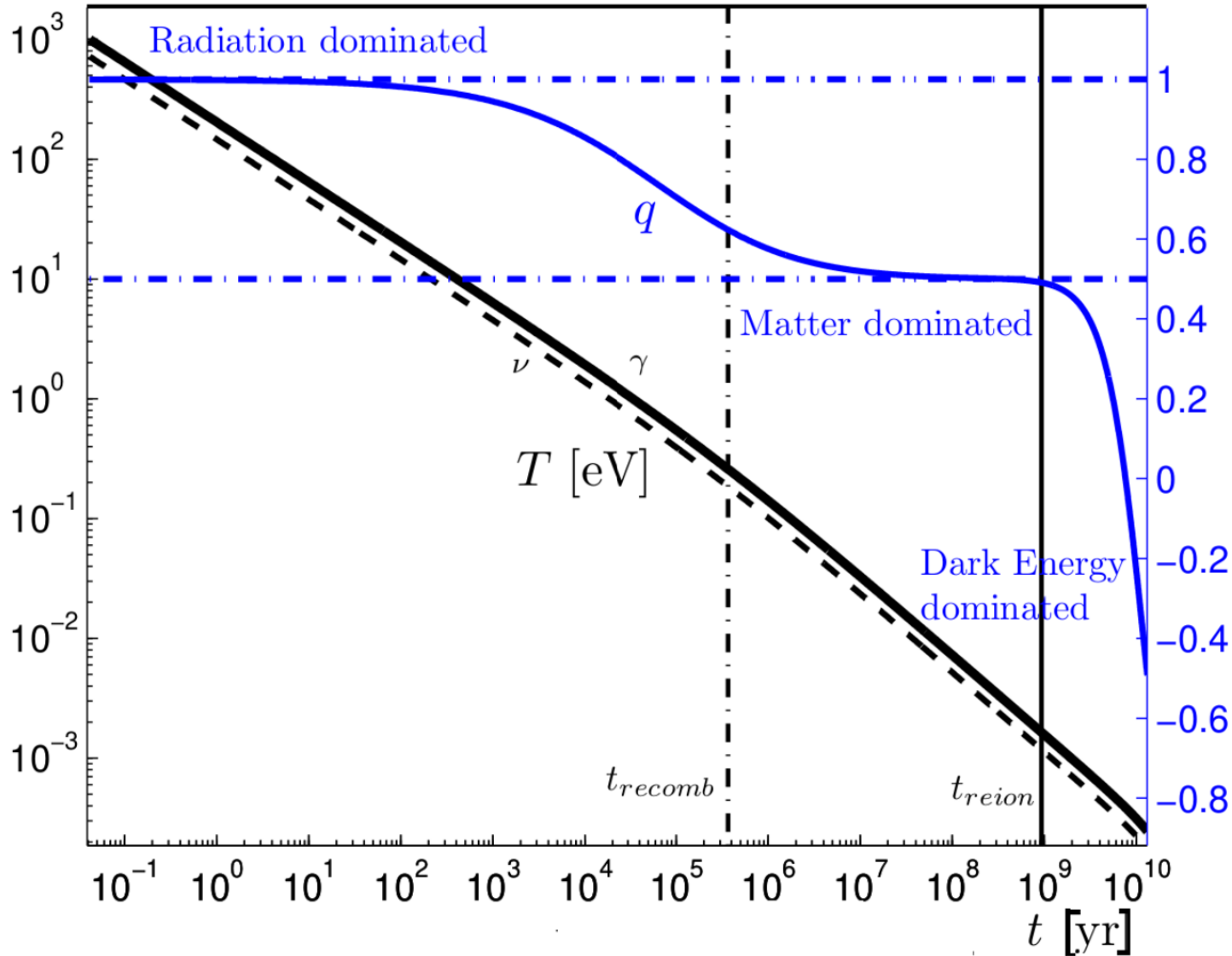


Shelbi J. Foster

Chemical equilibrium: A quick look at the Universe's composition



Cosmological eras of the Universe



Hubble parameter: $H(t) \equiv \frac{\dot{a}}{a}$

Deceleration: $q(t) \equiv -\frac{a\ddot{a}}{\dot{a}^2}$

Hubble dynamics: $\dot{H} = -H^2(1 + q)$

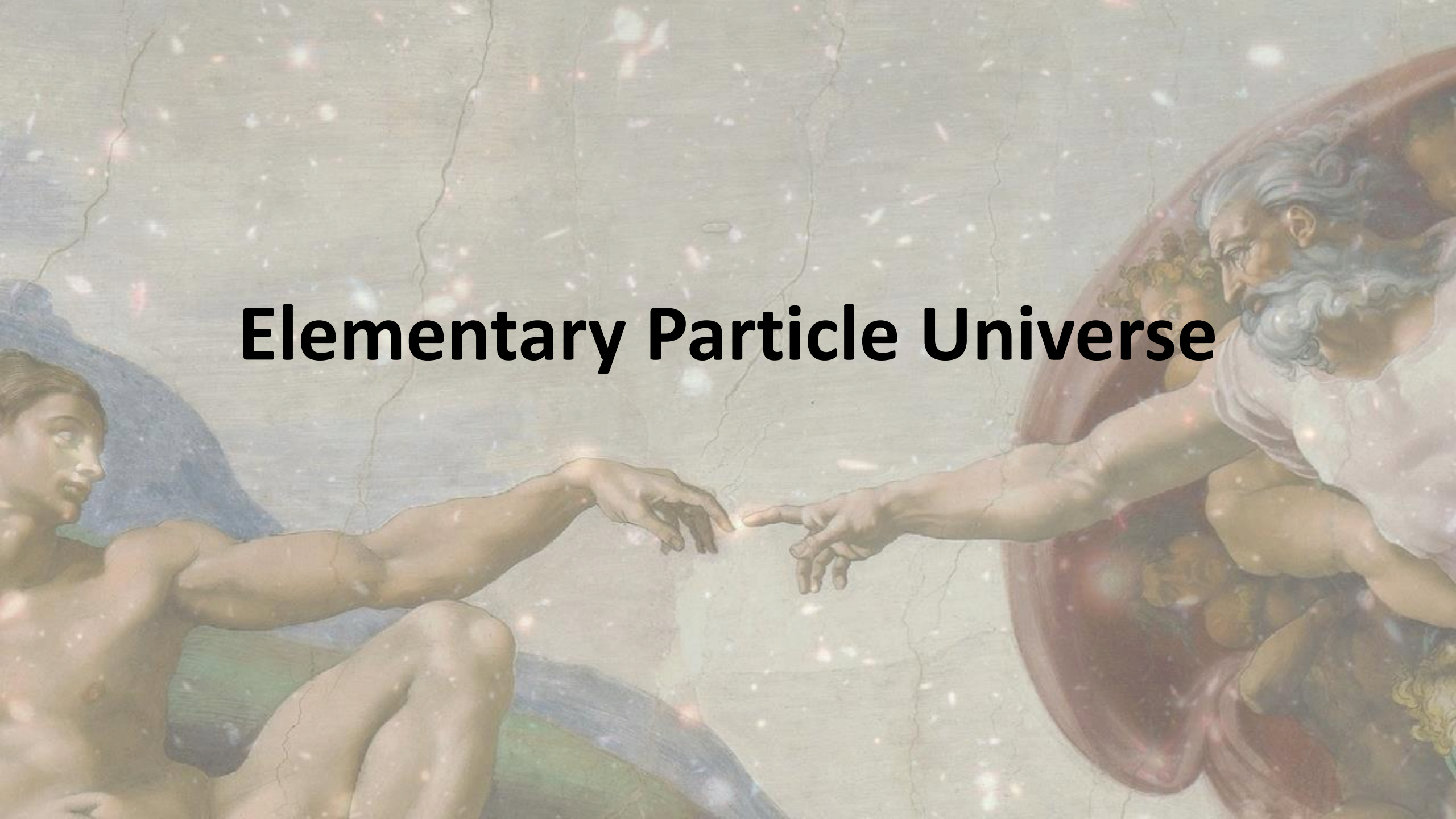
We can write both H and q in terms of the pressure P and density ρ of the universe.

$$H^2 = \frac{\rho}{3M_P^2} \quad q = \frac{1}{2} \left(1 + 3 \frac{P}{\rho} \right)$$

This lets us integrate to obtain the dynamical evolution of the universe in terms of composition.

How deceleration “q” evolved

Elementary Particle Universe



Overview of primordial cosmic plasma

After the “Big Bang,” the universe was a high temperature and density fireball.

Primordial quark-gluon plasma (the standard model of particle physics epoch):

At the early time when $130\text{GeV} > T > 150\text{MeV}$, the standard model particles define the property and evolution of the Universe

RHI
(experiments probe here)

Hadronic matter plasma epoch begins:

After hadronization quarks and mesons become confined within baryon and mesons, the visible matter in the Universe.

CB ν
(future experiments probe here)

Lepton-photon plasma emerges as dominant energy content:

For temperature $10\text{ MeV} > T > 2\text{ MeV}$, the Universe contained relativistic electrons, positrons, photons, and three species of (anti)neutrinos.

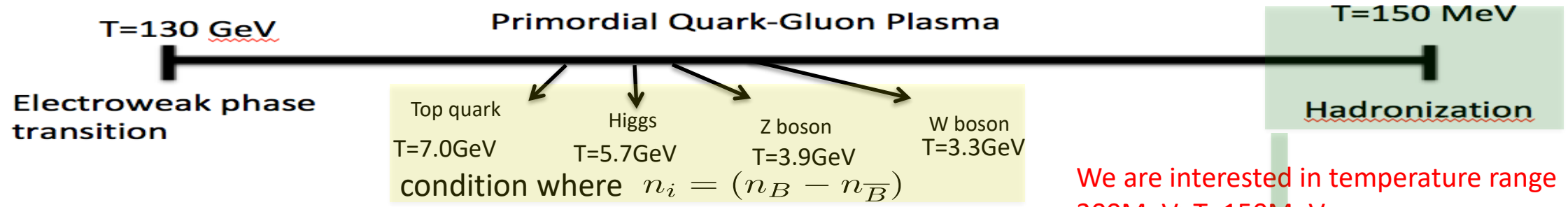
BBN here
(observables)

Electron-positron plasma persists:

After neutrinos freeze-out, the cosmic plasma was dominated by electrons, positrons, and photons. Positrons existed until $T=0.02\text{MeV}$.

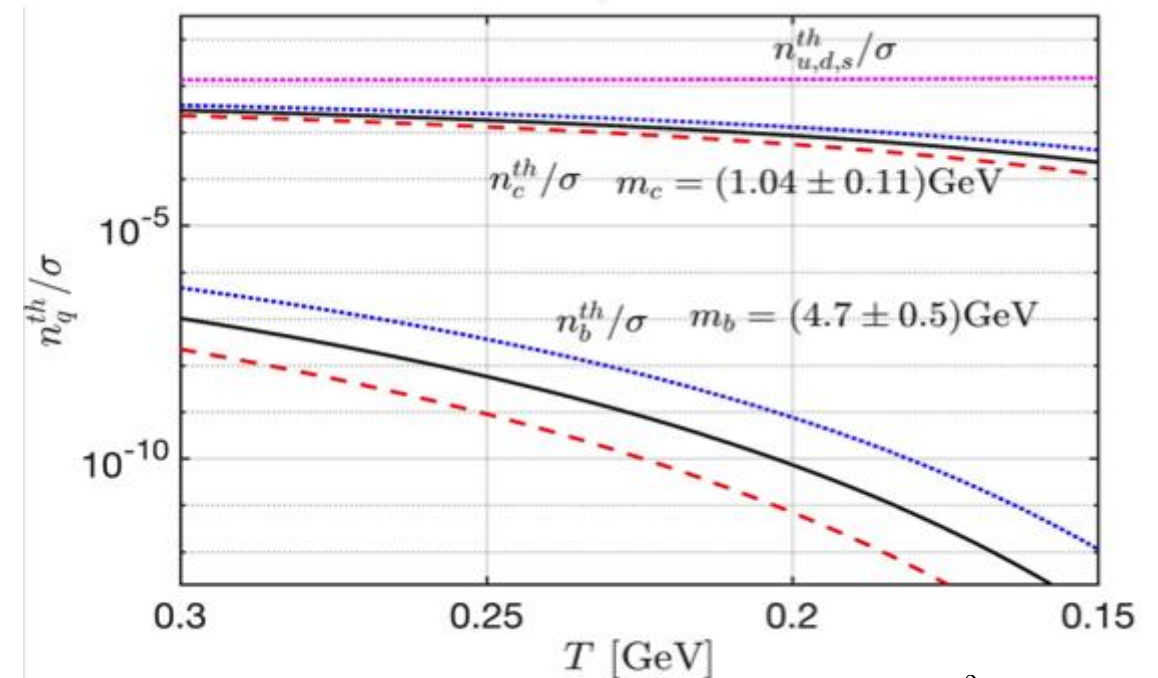
The SM Universe: Primordial Quark-Gluon Plasma 130GeV to 150MeV

The primordial QGP epoch begins after the electroweak phase transition and ends when the universe reach the hadronization.



$\approx 2.3 \text{ MeV}/c^2$ mass 2/3 charge 1/2 spin u up	0.511 MeV/c ² -1 charge 1/2 spin e electron	$\approx 1.275 \text{ GeV}/c^2$ 2/3 charge 1/2 spin c charm	105.7 MeV/c ² -1 charge 1/2 spin μ muon	$\approx 173.07 \text{ GeV}/c^2$ 2/3 charge 1/2 spin t top	1.777 GeV/c ² -1 charge 1/2 spin τ tau
$\approx 4.8 \text{ MeV}/c^2$ -1/3 charge 1/2 spin d down	$< 2.2 \text{ eV}/c^2$ 0 charge 1/2 spin ν_e electron neutrino	$\approx 95 \text{ MeV}/c^2$ -1/3 charge 1/2 spin s strange	$< 0.17 \text{ MeV}/c^2$ 0 charge 1/2 spin ν_μ muon neutrino	$\approx 4.18 \text{ GeV}/c^2$ -1/3 charge 1/2 spin b bottom	$< 15.5 \text{ MeV}/c^2$ 0 charge 1/2 spin ν_τ tau neutrino
0 charge 0 spin γ photon	91.2 GeV/c ² 0 charge 1 spin Z Z boson	80.4 GeV/c ² ±1 charge 1 spin W W boson	0 charge 0 spin g gluon	QUARKS LEPTONS GAUGE BOSONS	$\approx 126 \text{ GeV}/c^2$ 0 charge 0 spin H Higgs boson

- Up/down are massless quarks and retain the equilibrium via gluon/quark fusion. Jean Letessier, JR, "Hadrons and Quark-Gluon Plasma"
- Strangeness retain the equilibrium via weak, electromagnetic, and strong interaction until T~13MeV
C.T. Yang and JR, "Cosmological strangeness abundance," Phys. Lett. B 827, 136944 (2022)
- We are working on the top quark, massive bosons W, Z and Higgs.



$$n_q = \frac{3}{4\pi^2} g_q \zeta(3) T^3 \quad n_{b,c}^{th} = \frac{g_{b,c}}{2\pi^2} T^3 \left(\frac{m_{b,c}}{T} \right)^2 K_2(m_{b,c}/T)$$

Chemical nonequilibrium: Bottom production/decay

In the primordial QGP, the bottom quarks can be produced via the strong interaction gluon/quark pair fusion processes and disappear via the weak interaction decay.



$\approx 4.18 \text{ GeV}/c^2$

$-1/3$

$1/2$

b

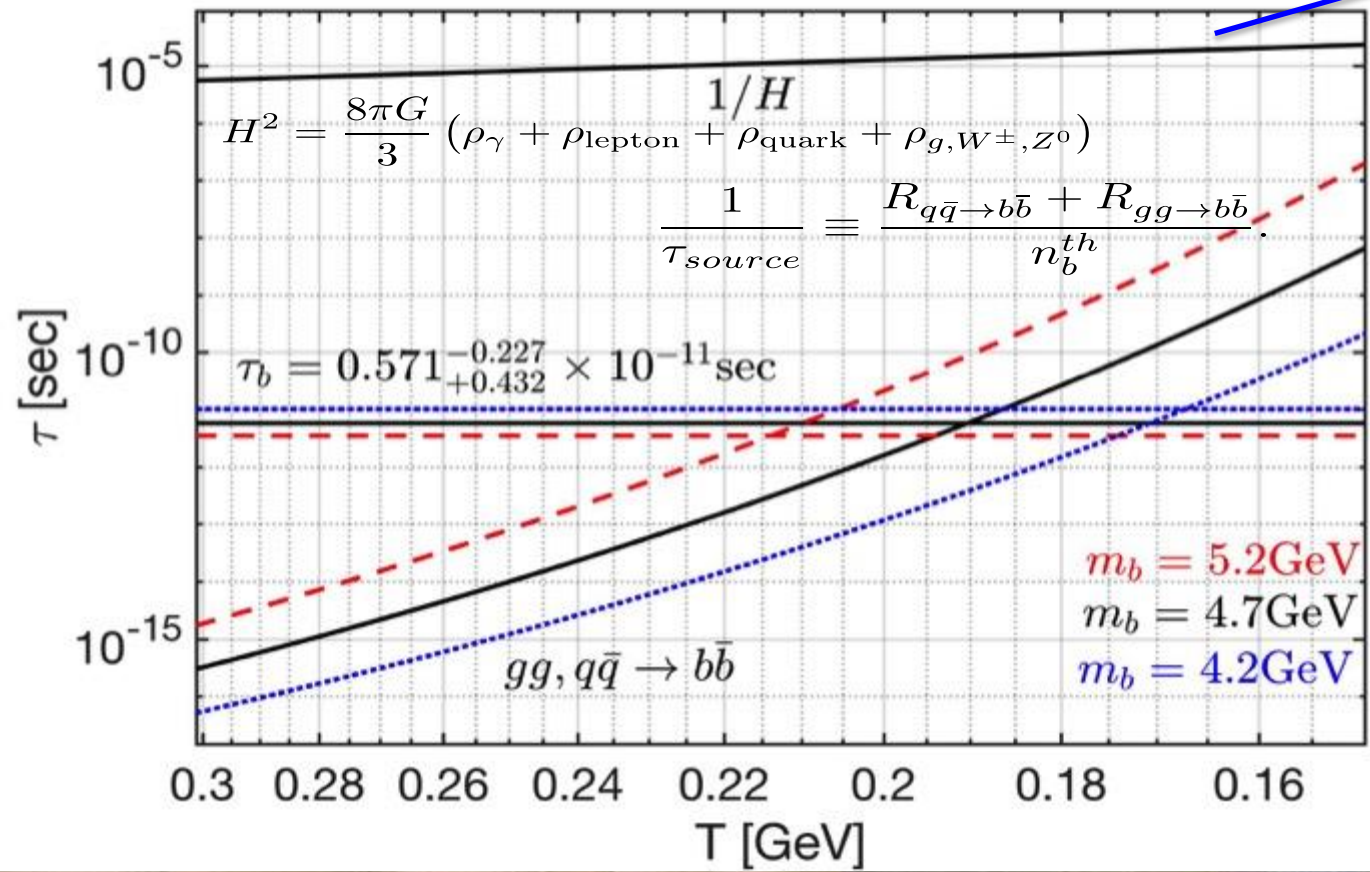
bottom

QUARKS

Assuming that three-body decay breaks detailed balance.

The universe expand 10^6 order slower compare to the bottom reaction at $T=200\text{MeV}$

The competition between decay and production reactions \rightarrow **Dynamic bottom abundance in QGP**



Number of bottom quark

Fugacity: the parameter to study nonequilibrium bottom

$$\frac{1}{V} \frac{dN_b}{dt} = (1 - \Upsilon_b^2) R_b^{\text{Source}} - \Upsilon_b R_b^{\text{Decay}}$$

Thermal decay rates per volume of bottom quark:
The bottom lifespan weighted with density of particles

$$R_b^{\text{Decay}} = \frac{dn_b/d\Upsilon_b}{\tau_b}$$

Kuznetsova, I., D. Habs, and JR (2008). **Pion and muon** production in e^- , e^+ , gamma plasma. Phys. Rev. D, 78, p. 014027.

Chemical nonequilibrium: Bottom Quark in QGP

Considering the **expansion of the universe** and the **number density of bottom quarks in Boltzmann limit**, the population equation for bottom quark can be written as

$$\frac{1}{V} \frac{dN_b}{dt} = (1 - \Upsilon_b^2) R_b^{\text{Source}} - \Upsilon_b R_b^{\text{Decay}}$$

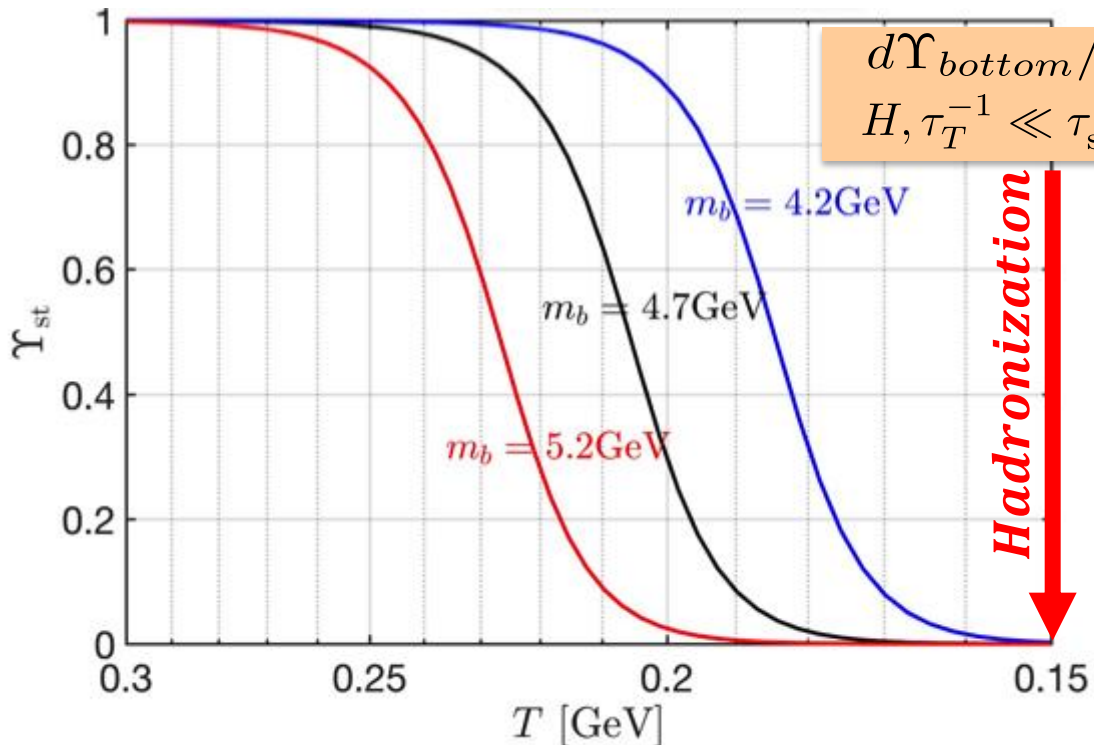
$$\frac{1}{V} \frac{dN_b}{dt} = \frac{dn_b}{d\Upsilon_b} \frac{d\Upsilon_b}{dt} + \frac{dn_b}{dT} \frac{dT}{dt} + 3Hn_b$$

$$n_b = \Upsilon_b \left[\frac{g_b}{2\pi^2} T^3 \left(\frac{m_b}{T} \right)^2 K_2(m_b/T) \right] = \Upsilon_b n_b^{\text{th}}$$

$$\frac{d\Upsilon_b}{dt} = (1 - \Upsilon_b^2) \frac{1}{\tau_b^{\text{Source}}} - \Upsilon_b \left(\frac{1}{\tau_b^{\text{Decay}}} + 3H - \frac{1}{\tau_T} \right)$$

$$\frac{1}{\tau_T} \equiv -\frac{dn_b^{\text{th}}/dT}{n_b^{\text{th}}} \frac{dT}{dt} \quad \frac{1}{\tau_{\text{source}}} \equiv \frac{R_{q\bar{q} \rightarrow b\bar{b}} + R_{gg \rightarrow b\bar{b}}}{n_b^{\text{th}}}$$

Considering the **dynamic equilibrium condition between production and decay reactions in adiabatic approximation**:



$$\frac{d\Upsilon_{\text{bottom}}}{dt} = 0$$

$$H, \tau_T^{-1} \ll \tau_{\text{source,decay}}^{-1}$$

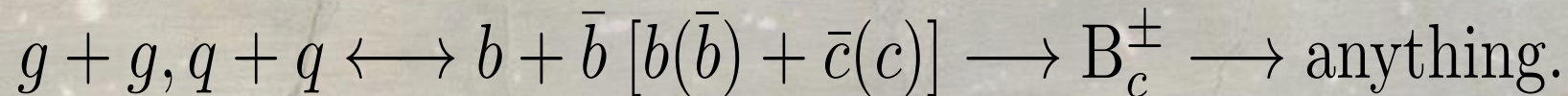
$$\Upsilon_{\text{st}} = \sqrt{1 + \left(\frac{\tau_{\text{source}}}{2\tau_{\text{decay}}} \right)^2 - \left(\frac{\tau_{\text{source}}}{2\tau_{\text{decay}}} \right)}$$

- The fugacity is a function of time because of the competition between the strong interaction production and WI decay
- Smaller mass of bottom quark allows nonequilibrium condition to move near to the transformation of QGP to HG phase.

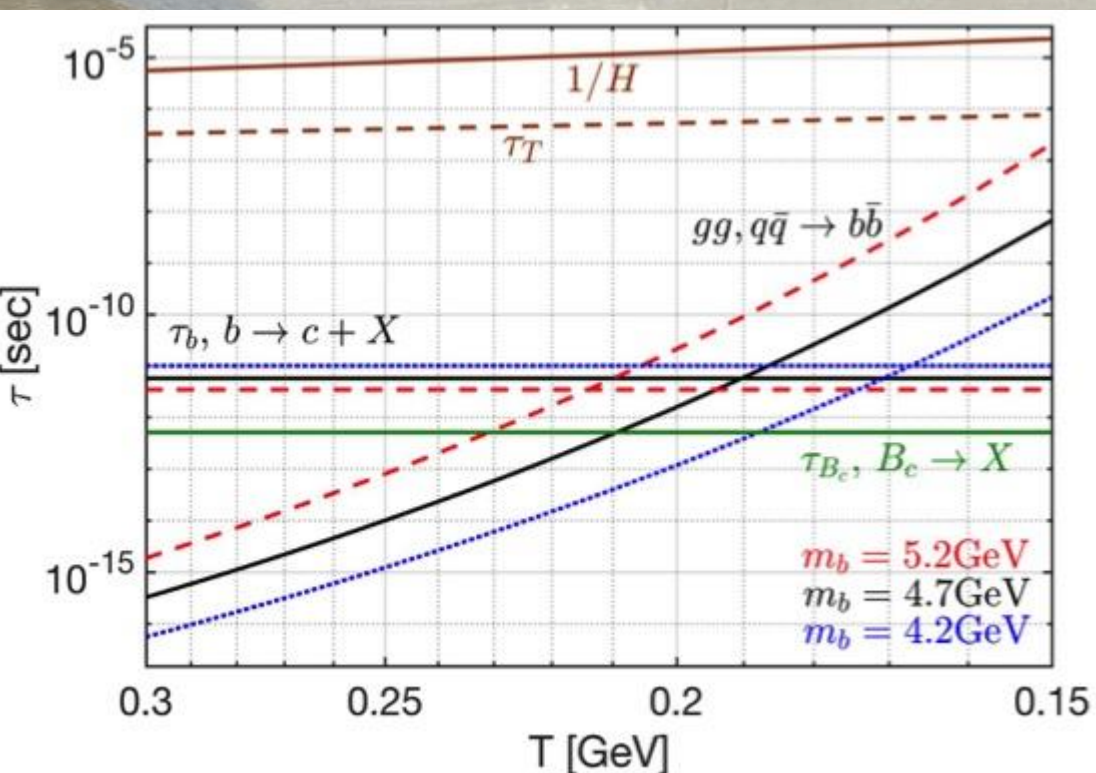
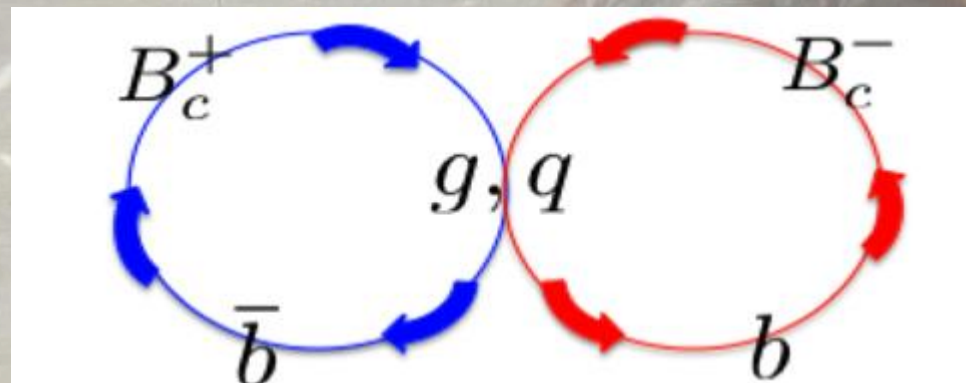
The bottom quark nonequilibrium occurs near to QGP – hadronization around the temperature $T = 0.3 \sim 0.15 \text{ GeV}$.

Nonequilibrium effects amplification: Urca cycle process

The off-equilibrium of bottom quark around the temperature windows $T = 0.3 \sim 0.15$ GeV is amplified by reaction cycling.



The rapid decay and reformation at picosecond scale assures that there are millions of individual microscopic processes involving bottom quark production and decay at the hadronization epoch of QGP \rightarrow **Urca process**



At low temperature, the number of bottom quark cycling can be estimated as

$$C_{\text{cycle}}|_{T=0.2\text{GeV}} = \frac{\tau_H}{\tau_{B_c}} \approx 2 \times 10^7$$

$$\tau_H = \frac{1}{H} = 1.272 \times 10^{-5}$$

$$\tau_{B_c} = 0.510 \times 10^{-12} \text{ sec}$$

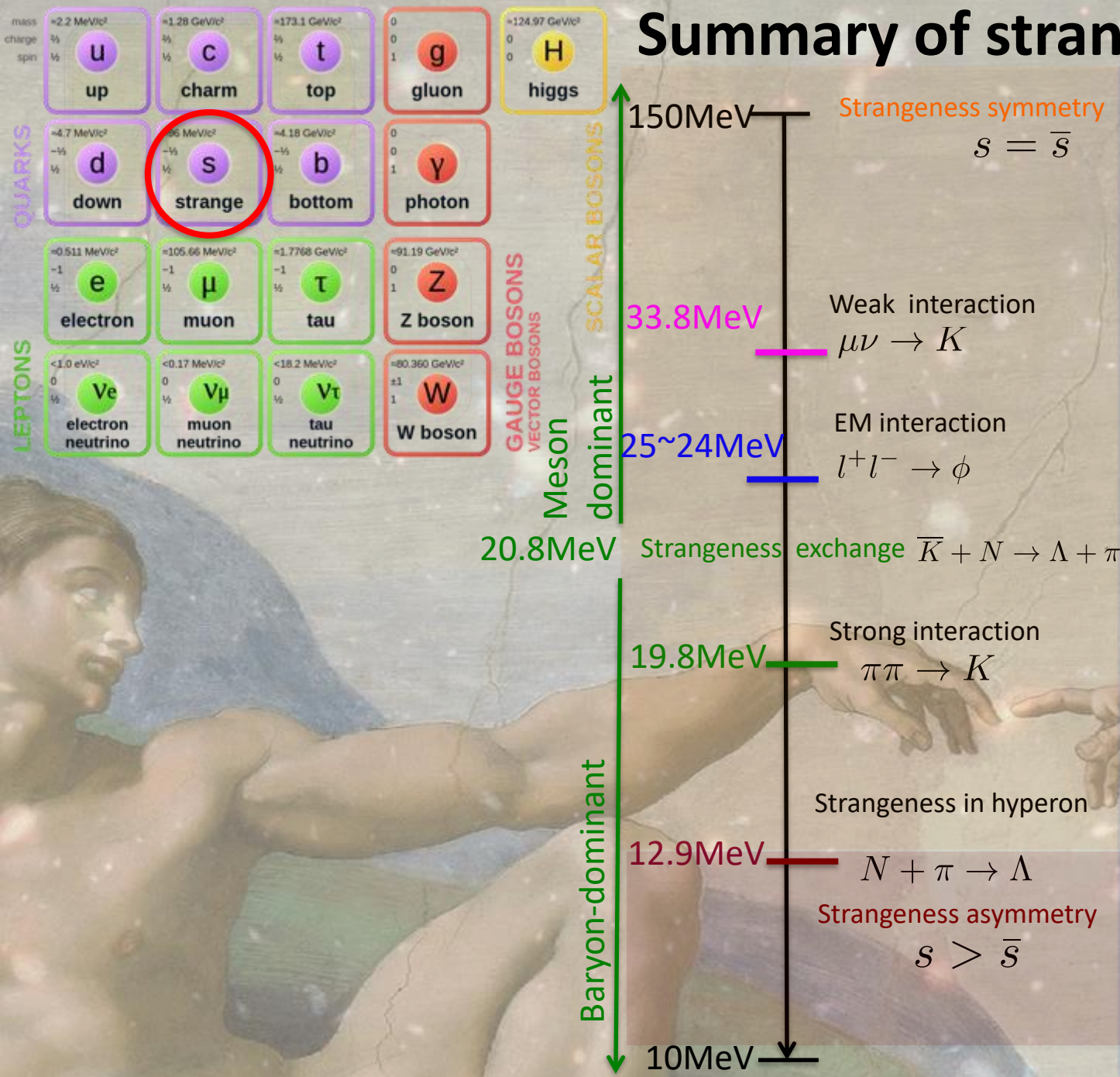
$$M_{bc} = 6274.5 \text{ MeV}$$

Any small and yet not observed violation of baryon number associated with bottom quarks can be amplified by the Urca process to required strength for today's observation

Hadronic Matter Universe



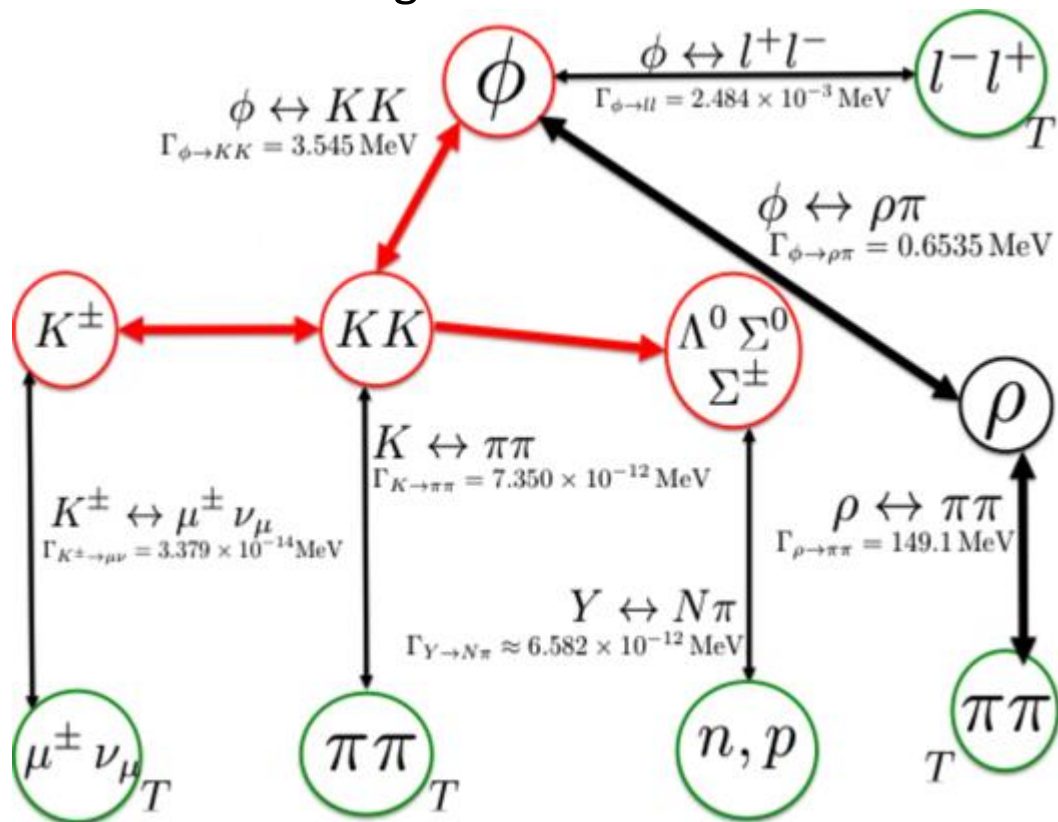
Summary of strangeness in the universe



- For temperature $T > 20.8$ MeV, the strangeness is predominantly present in the mesons with $s = \bar{s}$
- Strange mesons can be produced by the inverse decay reactions are in equilibrium via weak, electromagnetic, and strong interactions in the primordial universe until $T=20$ MeV
- Below the temperature $T < 20$ MeV, all the detail balances in the strange meson reactions are broken and the strangeness in the meson sector should disappear rapidly
- For temperature $20.8 > T > 12.8$ MeV, the strangeness is predominantly present in the hyperons and anti-strangeness in kaon, then keep $s = \bar{s}$
- For $T < 12.9$ MeV the reaction $K + N \rightarrow \Lambda + \pi$ becomes slower than the strangeness decay $N + \pi \leftrightarrow \Lambda$, and we have $s > \bar{s}$

Strangeness network reactions

The chemical non-equilibrium can be achieved by breaking the detailed balance between particle production reaction and decay processes. The relevant reactions following QGP hadronization in the temperature interval $150 > T > 10$ MeV range.



Strangeness in mesons:

$$\begin{aligned} \pi + \pi &\leftrightarrow K, & \mu^\pm + \nu &\leftrightarrow K^\pm \\ l^+ + l^- &\leftrightarrow \phi, & \rho + \pi &\leftrightarrow \phi, \\ \pi + \pi &\leftrightarrow \rho \end{aligned}$$

Competition between Hubble and production

Strangeness in hyperons:

$$\begin{aligned} \pi + N &\leftrightarrow K + \Lambda, \\ K + N &\leftrightarrow \Lambda + \pi, \\ \Lambda &\leftrightarrow N + \pi \end{aligned}$$

Competition between production and decay

Pions retain their chemical equilibrium until $T=3\sim 5$ MeV

I. Kuznetsova, D.Habs and JR. Phys. Rev. D 78, 014027 (2008)

Muons retain their chemical equilibrium until $T = 4.135$ MeV.

JR and C. T. Yang (2021). The muon abundance in the primordial Universe. Acta Phys. Polon. B, 52, p. 277.

Chemical nonequilibrium: partial freezeout due to Hubble expansion

Once the reactions decouple from the cosmic plasma, the corresponding detailed balance can be broken and the **inverse decay reactions are acting like a "hole" in the abundance "pot"**.

Relaxation time $\tau_{12 \rightarrow 3} \equiv \frac{n_1^{eq}}{R_{12 \rightarrow 3}}$

P.Koch, B.Muller and JR, 'Strangeness in Relativistic Heavy Ion Collisions', Phys. Rept. 142 167-262 (1986)

Freezeout Condition $\tau_{12 \rightarrow 3}(T_f) = 1/H(T_f)$

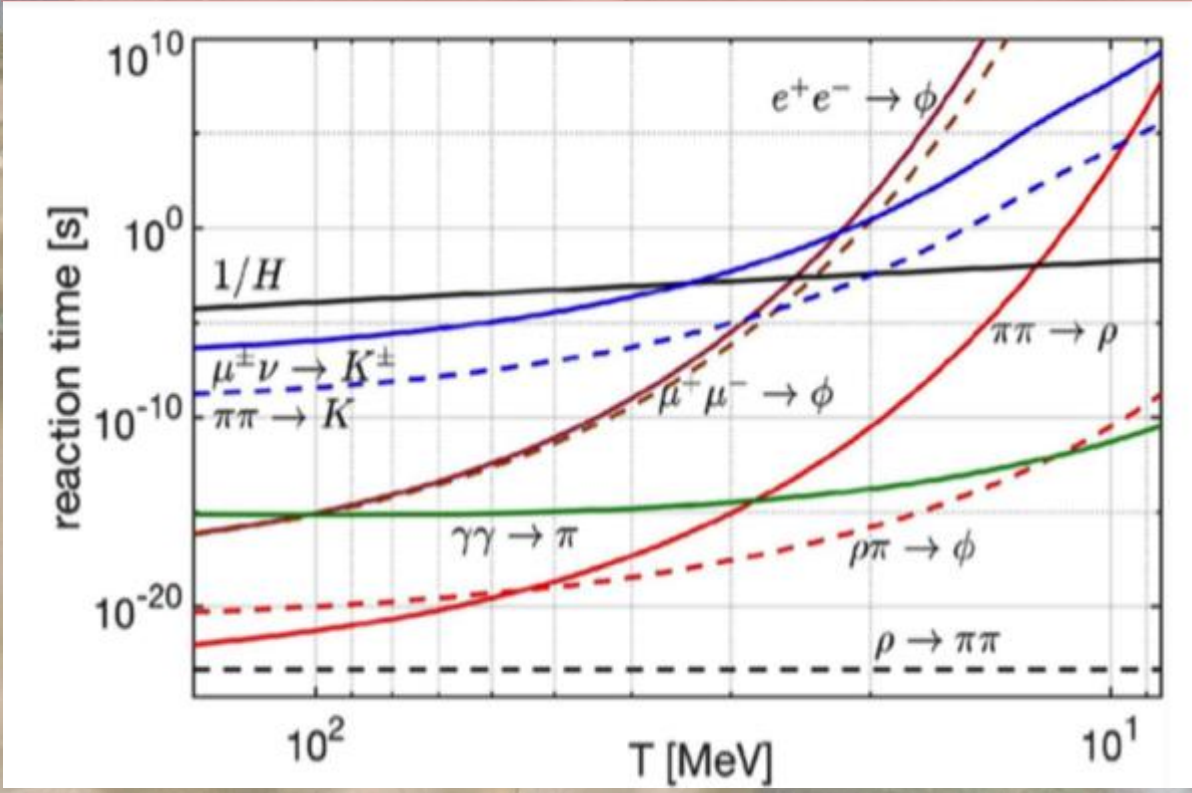
$$H^2 = H_{rad}^2 \left(1 + \frac{\rho_{\pi, \mu, \rho}}{\rho_{rad}} + \frac{\rho_{strange}}{\rho_{rad}} \right) \quad H_{rad}^2 = \frac{8\pi G_N}{3} (\rho_\gamma + \rho_\nu + \rho_{e^\pm})$$


$$\frac{1}{\Delta T_f} \equiv \left[\frac{1}{(\Gamma_{12 \rightarrow 3}/H)} \frac{d(\Gamma_{12 \rightarrow 3}/H)}{dT} \right]_{T_f}, \quad \Gamma_{12 \rightarrow 3} \equiv \frac{1}{\tau_{12 \rightarrow 3}}$$

Reactions	Freezeout Temperature (MeV)	ΔT_f (MeV)
$\mu^\pm \nu \rightarrow K^\pm$	$T_f = 33.8$ MeV	3.5 MeV
$e^+ e^- \rightarrow \phi$	$T_f = 24.9$ MeV	0.6 MeV
$\mu^+ \mu^- \rightarrow \phi$	$T_f = 23.5$ MeV	0.6 MeV
$\pi\pi \rightarrow K$	$T_f = 19.8$ MeV	1.2 MeV
$\pi\pi \rightarrow \rho$	$T_f = 12.3$ MeV	0.2 MeV

Most of ρ -mesons decay faster before they can convert to ϕ -meson: Strangeness nonequilibrium

Below the temperature $T < 20$ MeV, all the detail balances in the strange meson reactions are broken.



The image is a digital composite of Michelangelo's 'The Creation of Adam' fresco. The background has been replaced with a dark, starry space scene filled with numerous white and pinkish star-like particles. The central text 'Lepton Universe' is overlaid in a bold, black, sans-serif font. The figures of Adam and God are visible on the left and right respectively, with their hands reaching toward each other in the center. The overall aesthetic is a blend of classical art and modern digital effects.

Lepton Universe

Accidental degeneracy of muon abundance with baryon abundance

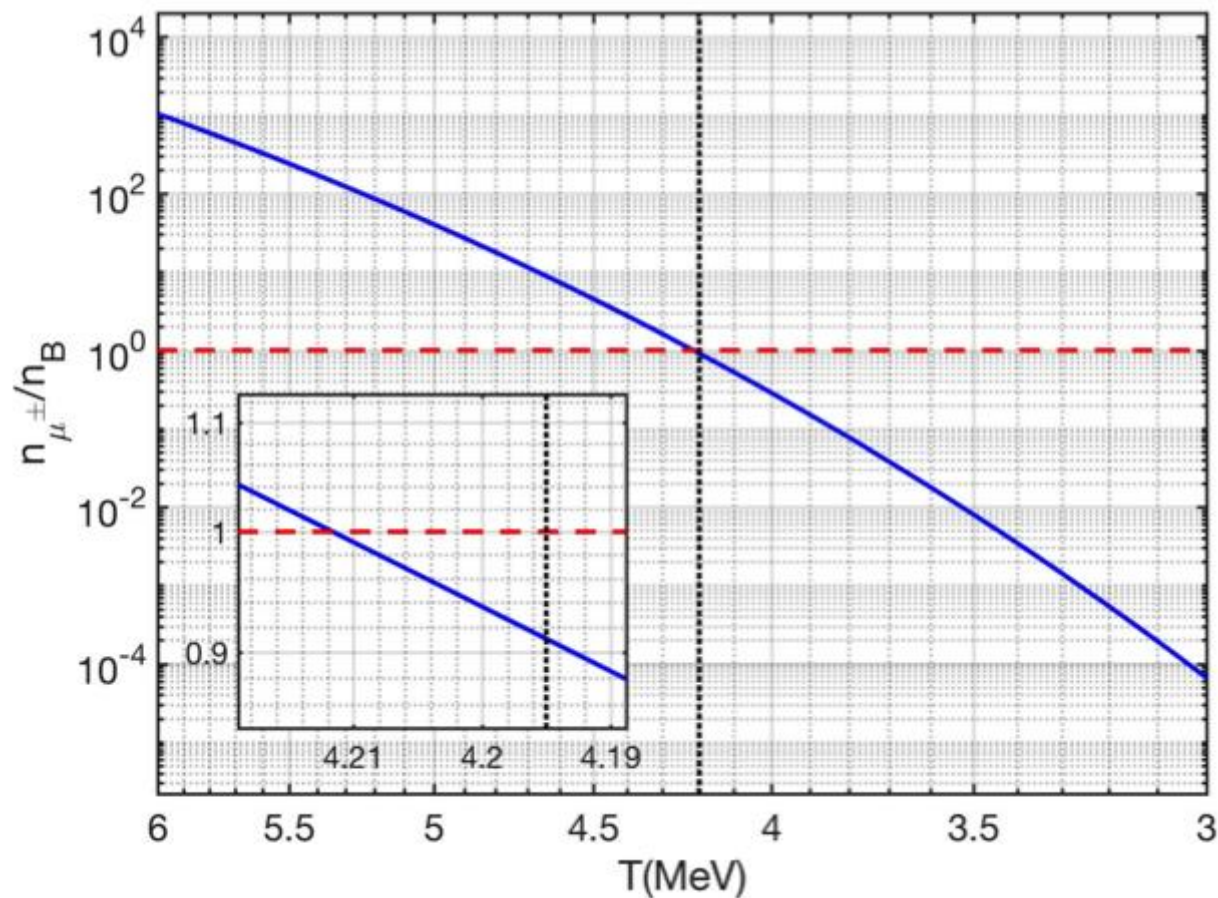
Using constant baryon-per-entropy ratio, the density between muon and baryon can be written as

$$\frac{n_{\mu^\pm}}{n_B} = \frac{n_{\mu^\pm}}{s} \frac{s}{n_B} = \frac{n_{\mu^\pm}}{s} \left(\frac{s_{\gamma e} (1 + s_\nu / s_{\gamma e})}{n_B} \right)_{t_0}$$

$$(n_B - n_{\bar{B}}) / n_\gamma = (0.609 \pm 0.06) \times 10^{-9} (\text{CMB})$$

$$s = \frac{2\pi^2}{45} g_*^s T^3$$

$$n_{\mu^\pm} = \frac{g_{\mu^\pm}}{2\pi^2} T^3 \left(\frac{m_\mu}{T} \right)^2 K_2(m_\mu/T)$$



Muon-baryon density ratio at persistence temperature:

$$T_\mu = 4.2 \text{ MeV}$$

$$n_{\mu^\pm} / n_B(T_\mu) \approx 0.911$$

The temperature for unity muon-to-baryon density ratio:

$$T_{\text{equal}} \approx 4.212 \text{ MeV}$$

$$n_{\mu^\pm} / n_B(T_{\text{equal}}) = 1$$

This means that the muon abundance could influence baryon evolution because muon number density is comparable to the baryon number density.

Long lasting muons could disappear just before neutrino freezeout coincidentally with the same abundance as baryon number

Boltzmann-Einstein Equation and Neutrino freeze-out

The Boltzmann equation describe the evolution of distribution function in phase space. In general, the Boltzmann equation in FLRW universe in the absence of electromagnetic field can be written as

$$\frac{\partial f}{\partial t} - \frac{(E^2 - m^2)}{E} H \frac{\partial f}{\partial E} = \frac{1}{E} \sum^g C_q[f]$$

As the Universe expanded, the interactions gradually became too slow to maintain equilibrium and particles freeze-out from the cosmic plasma.

Freeze-out condition $\tau_{rel}^{-1} \leq H$ $\tau_{rel}^{-1} = \frac{g}{(2\pi)^3 n} \int d^3p \frac{1}{E} C_q[f]$

The freeze-out process involves several steps that lead to the free streaming

Extension of the textbook knowledge

Chemical freeze-out :

Freeze-out of inelastic scattering

Kinetic freeze-out :

Freeze-out of elastic scattering

Free-streaming:

Particles had fully decoupled from the cosmic plasma

Prior to the chemical freeze-out :

$$f_{th} = \frac{1}{\exp((E - \mu)/T) + 1}$$

Between chemical and kinetic freeze-out

$$f_k = \frac{1}{\Upsilon^{-1} \exp((E - \mu)/T) + 1}$$

Fugacity : controlling overall abundance of given particles and antiparticle

After kinetic freeze-out, free-streaming distribution

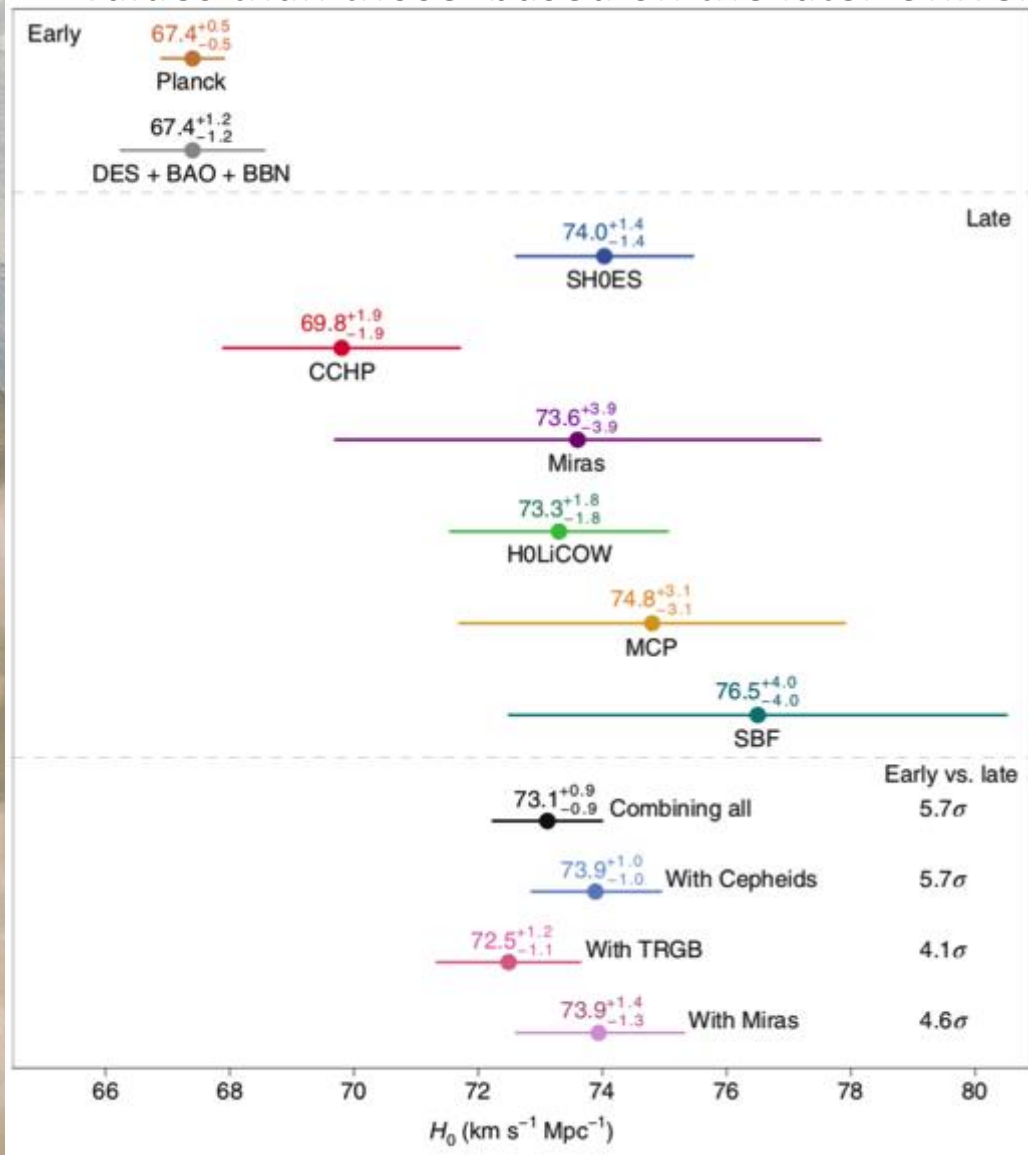
$$f_{fs} = \frac{1}{\Upsilon_f^{-1} \exp\left(\sqrt{\frac{E^2 - m^2}{T_{fs}^2} + \frac{m^2}{T_f^2}} - \frac{\mu}{T_f}\right) + 1}$$

Effective free-streaming temperature

$$T_{fs} \equiv \frac{\alpha(t_f)}{\alpha(t)} T_f$$

Hubble tensions between the primordial and current Universe

Estimates of the universe's expansion rate based on physics of the primordial Universe tend to have lower values than those based on the later Universe.



After neutrinos decoupled from the cosmic plasma and become free-streaming, it still continues to play a significant role in the evolution of the Universe

$$H^2 = \frac{8\pi G_N}{3} [\rho_{\text{matter}} + \rho_\gamma + \rho_\nu] + \frac{\Lambda}{3}$$

$$= \frac{8\pi G_N}{3} \left[\rho_{\text{matter}} + \rho_\gamma + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_\nu^{\text{eff}} \rho_\gamma \right] + \frac{\Lambda}{3}$$

Effective number of neutrinos related to observed Universe dynamics

$$N_\nu^{\text{eff}} \equiv \frac{\rho_\nu^{\text{tot}}}{\frac{7}{8} \rho_\gamma \left(\frac{4}{11} \right)^{4/3}} = \frac{\rho_\nu^{\text{tot}}}{\frac{7\pi^2}{120} \left(\frac{4}{11} \right)^{4/3} T_\gamma^4}$$



Dr. Cheng Tao Yang



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Neutrino Freeze-out in the primordial Universe

For temperature $4\text{MeV} > T > 0.02 \text{ MeV}$, the Universe contained relativistic electrons, positrons, photons, and three species of (anti)neutrinos

$$\text{Cosmic plasma: } \{ \gamma, e^{\pm}, \nu_l, \bar{\nu}_l \}$$

Neutrino freeze-out temperature:

The freeze-out temperature impact the energy and entropy transfer from electron-positron annihilation to neutrinos

→ impact N_{ν}^{eff}

Effective number of neutrino

$$N_{\nu}^{\text{eff}} \equiv \frac{\rho_{\nu}^{\text{tot}}}{\frac{7}{8} \rho_{\gamma} \left(\frac{4}{11}\right)^{4/3}} = \frac{\rho_{\nu}^{\text{tot}}}{\frac{7\pi^2}{120} \left(\frac{4}{11}\right)^{4/3} T_{\gamma}^4}$$

The Planck collaboration 2013 result:

$$N_{\nu}^{\text{eff}} = 3.36 \pm 0.34 \text{ (CMB)}$$

$$N_{\nu}^{\text{eff}} = 3.62 \pm 0.25 \text{ (CMB} + H_0)$$

Ade, P. A. R. et al. (2014). Planck 2013 results. XVI.. Astron. Astrophys., 571, p. A16.

The Planck collaboration 2018 result:

$$N_{\nu}^{\text{eff}} = 2.99 \pm 0.17$$

In contradiction to present day
Universe dynamics

N.Aghanim, et al., "Planck 2018 results. VI. Cosmological parameters," Astron. Astrophys. 641, A6 (2020)



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Relic neutrino freeze-out: Dependence on natural constants

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Abstract

Analysis of cosmic microwave background radiation fluctuations favors an effective number of neutrinos, $N_{\nu} > 3$. This motivates a reinvestigation of the neutrino freeze-out process. Here we characterize the dependence of N_{ν} on the Standard Model (SM) parameters that govern neutrino freeze-out. We show that N_{ν} depends on a combination η of several natural constants characterizing the relative strength of weak interaction processes in the early Universe and on the Weinberg angle $\sin^2 \theta_W$. We determine numerically the dependence $N_{\nu}(\eta, \sin^2 \theta_W)$ and discuss these results. The extensive numerical computations are made possible by two novel numerical procedures: a spectral method Boltzmann equation solver adapted to allow for strong reheating and emergent chemical non-equilibrium, and a method to evaluate Boltzmann equation collision integrals that generates a smooth integrand.

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Neutrino freeze-out temperature

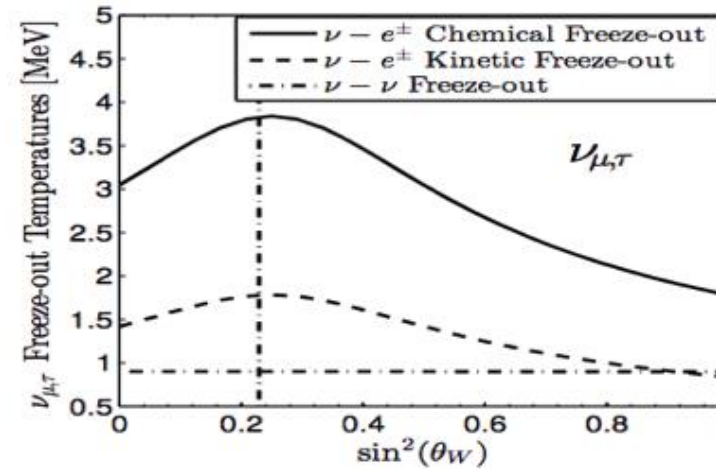
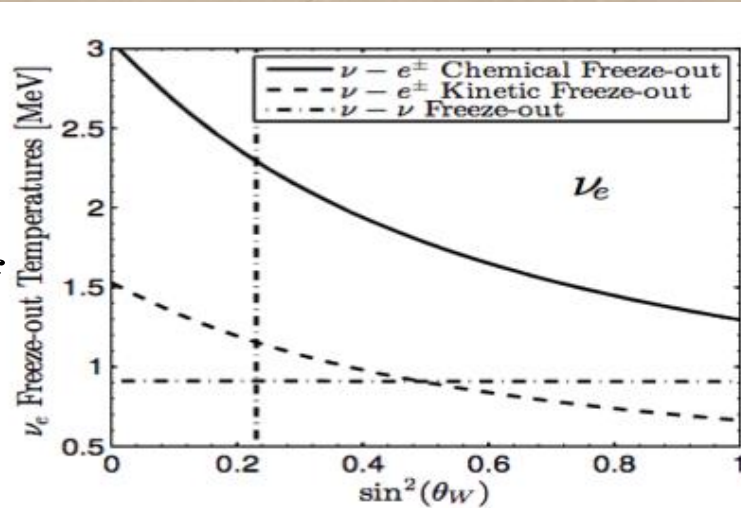
We show that the neutrino freeze-out temperature is controlled by standard model parameters which can be modified in high temperature or plasma environment.

SM parameters: Weinberg angle
 $\sin^2 \theta_W = 0.231$

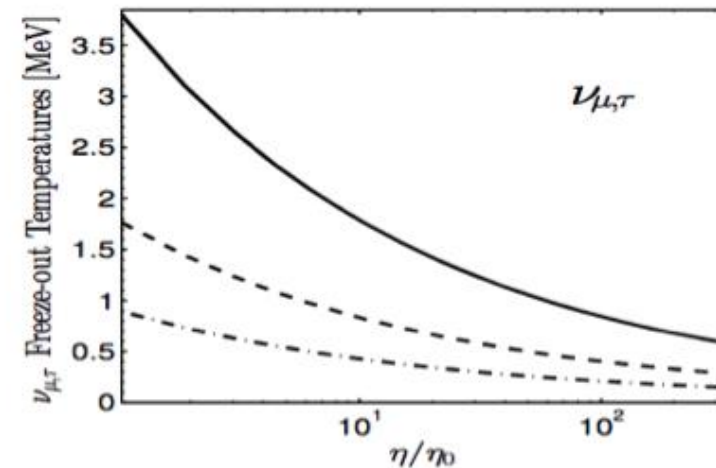
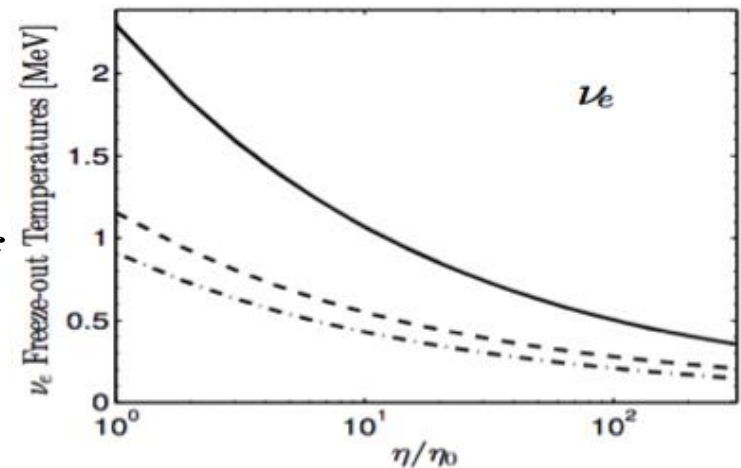
Interaction strength parameter

$$\eta \equiv M_p m_e^3 G_F^2 \quad \eta_0 \equiv M_p m_e^3 G_F^2|_0 = 0.04421$$

T_f



T_f



Magnetism in primordial cosmology

PHYSICAL REVIEW D **108**, 123522 (2023)

Matter-antimatter origin of cosmic magnetism

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(Received 30 August 2023; accepted 15 November 2023; published 11 December 2023)

We explore the hypothesis that the abundant presence of relativistic antimatter (positrons) in the primordial Universe is the source of the intergalactic magnetic fields we observe in the Universe today. We evaluate both Landau diamagnetic and magnetic dipole moment paramagnetic properties of the very dense primordial electron-positron e^+e^- -plasma, and obtain in quantitative terms the relatively small magnitude of the e^+e^- magnetic moment polarization asymmetry required to produce a consistent self-magnetization in the Universe.

[Steinmetz, Andrew, Cheng Tao Yang, and Johann Rafelski. "Matter-antimatter origin of cosmic magnetism." Physical Review D 108.12 \(2023\): 123522.](#)



universe For Remo Ruffini Festschrift

Review

A Short Survey of Matter-Antimatter Evolution in the Primordial Universe

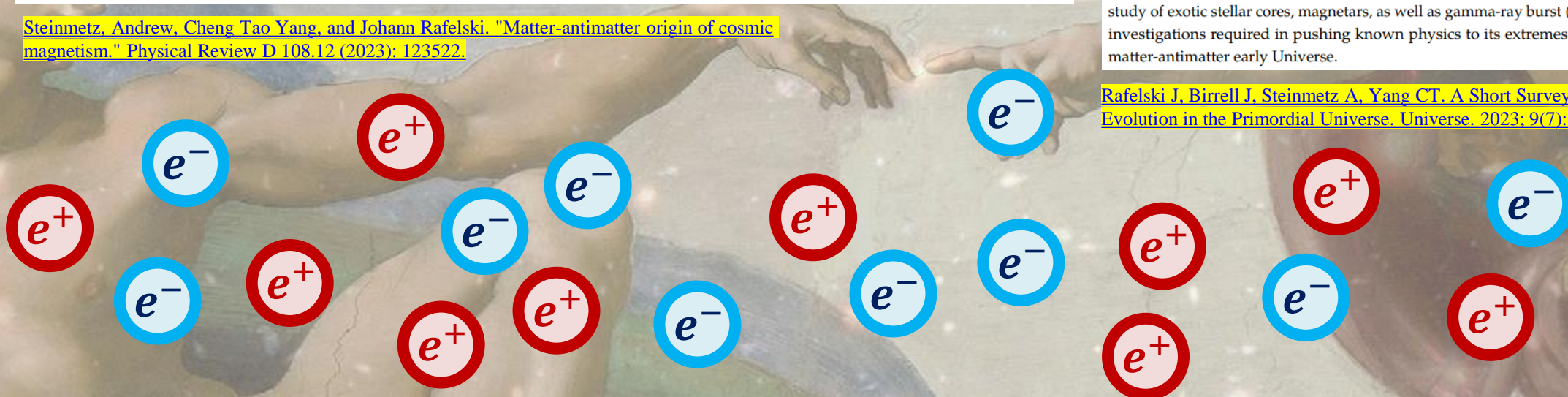
Johann Rafelski^{1*}, Jeremiah Birrell¹, Andrew Steinmetz¹ and Cheng Tao Yang¹

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* Correspondence: johannr@arizona.edu

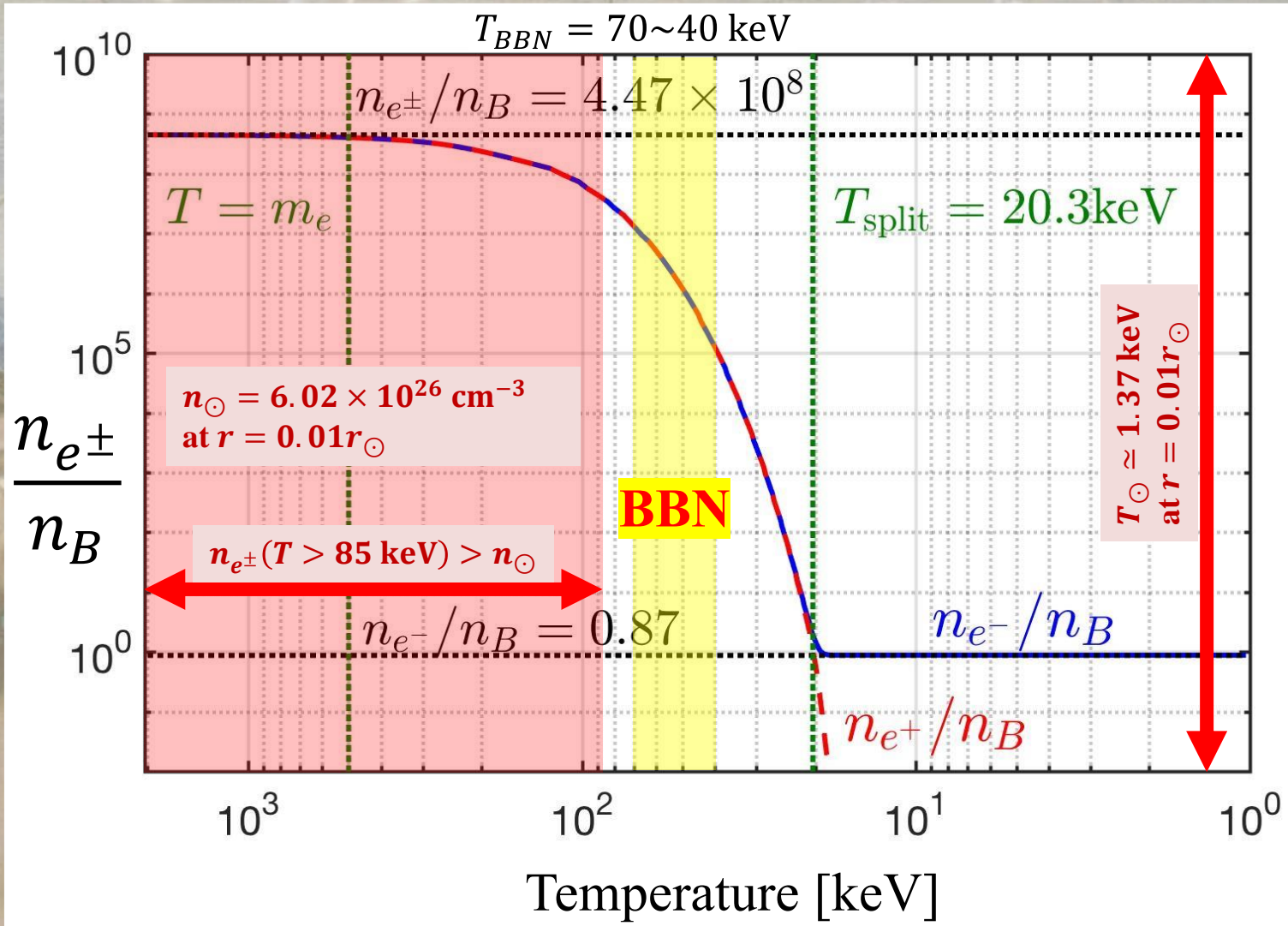
Abstract: We offer a survey of the matter-antimatter evolution within the primordial Universe. While the origin of the tiny matter-antimatter asymmetry has remained one of the big questions in modern cosmology, antimatter itself has played a large role for much of the Universe's early history. In our study of the evolution of the Universe we adopt the position of the standard model Lambda-CDM Universe implementing the known baryonic asymmetry. We present the composition of the Universe across its temperature history while emphasizing the epochs where antimatter content is essential to our understanding. Special topics we address include the heavy quarks in quark-gluon plasma (QGP), the creation of matter from QGP, the free-streaming of the neutrinos, the vanishing of the muons, the magnetism in the electron-positron cosmos, and a better understanding of the environment of the Big Bang Nucleosynthesis (BBN) producing the light elements. We suggest but do not explore further that the methods used in exploring the early Universe may also provide new insights in the study of exotic stellar cores, magnetars, as well as gamma-ray burst (GRB) events. We describe future investigations required in pushing known physics to its extremes in the unique laboratory of the matter-antimatter early Universe.

[Rafelski J, Birrell J, Steinmetz A, Yang CT. A Short Survey of Matter-Antimatter Evolution in the Primordial Universe. Universe. 2023; 9\(7\):309.](#)



e^+e^- density compared to solar core density

Above $T \simeq 85$ keV, the density of the e^+e^- plasma exceeds that of the core of the sun.



Important point

The primordial cosmic environment was very different than the plasma present inside the cores of stars like our Sun.

The plasma of BBN is less dense but far hotter than that of the Sun's core. The plasma prior to BBN is both denser and hotter.

Solar burning occurs with 1 electron per proton for screening while BBN occurs with many thousands of electrons (positrons) for screening.

For further reading on e^+e^- plasma and BBN see **Grayson, et. al. 2022.**

Grayson, C., et al. "Dynamic magnetic response of the quark-gluon plasma to electromagnetic fields." *Physical Review D* 106.1 (2022): 014011.

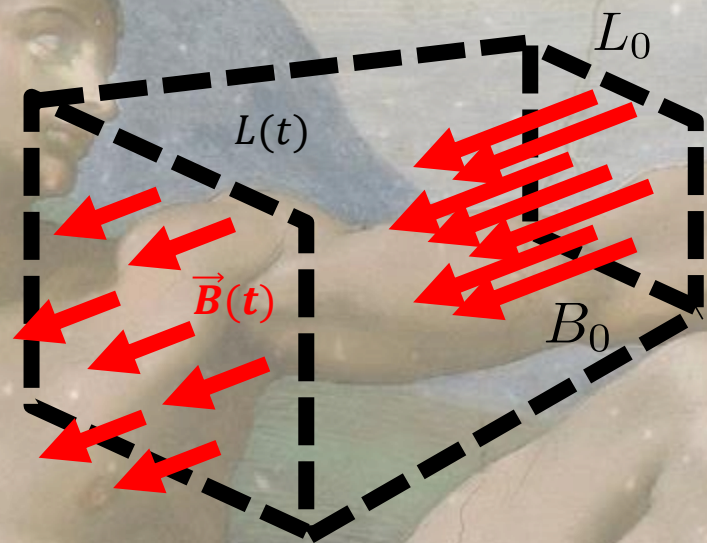
How magnetism is function of temperature

Magnetic flux is conserved over a comoving surface

$$L(t)^2 = L_0^2 a(t)^2 / a(t_0)^2$$

The magnetic field then dilutes over time as

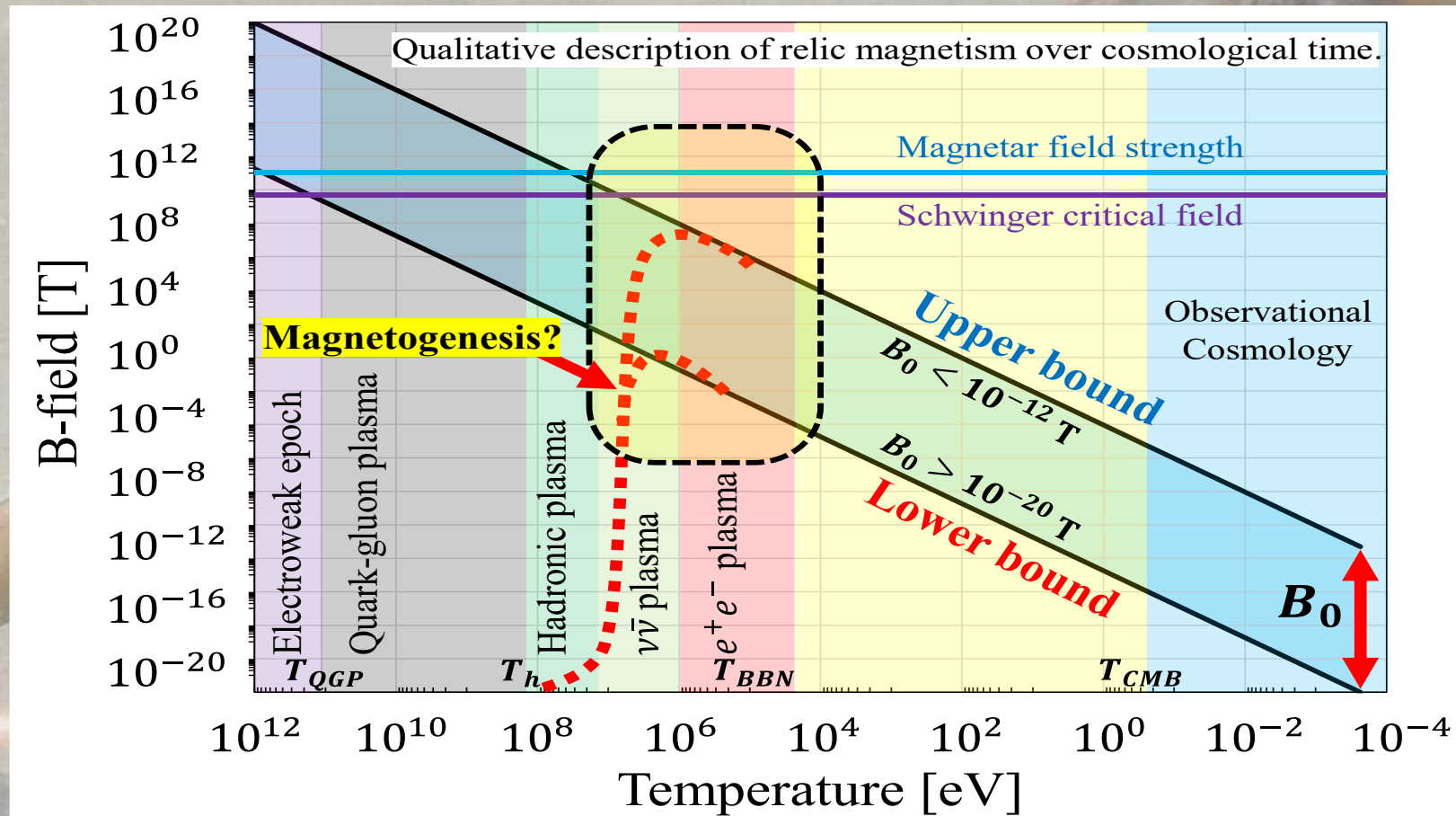
$$B(t)^2 = B_0^2 a(t_0)^2 / a(t)^2$$



Magnetic flux:

$$\Phi_B = B(t)L^2(t) = B_0L_0^2$$

Cosmic relic magnetism – Pre-CMB signal



The upper and lower bounds on intergalactic magnetic fields (IGMF) coherent over Mpc scales is shown.

Magnetization of e⁻e⁺-plasma in Boltzmann limit

The partition function of nonrelativistic electron-positron plasma with a uniform magnetic field pointing along the z-axis

$$\ln \mathcal{Z}_{e^+e^-} = \frac{T^3 V}{2\pi^2} \left[2 \cosh \left(\frac{\mu_e}{T} \right) \right] \sum_{i=\pm} \left\{ x_i^2 K_2(x_i) + \frac{b_0}{2} x_i K_1(x_i) + \frac{b_0^2}{12} K_0(x_i) \right\}$$

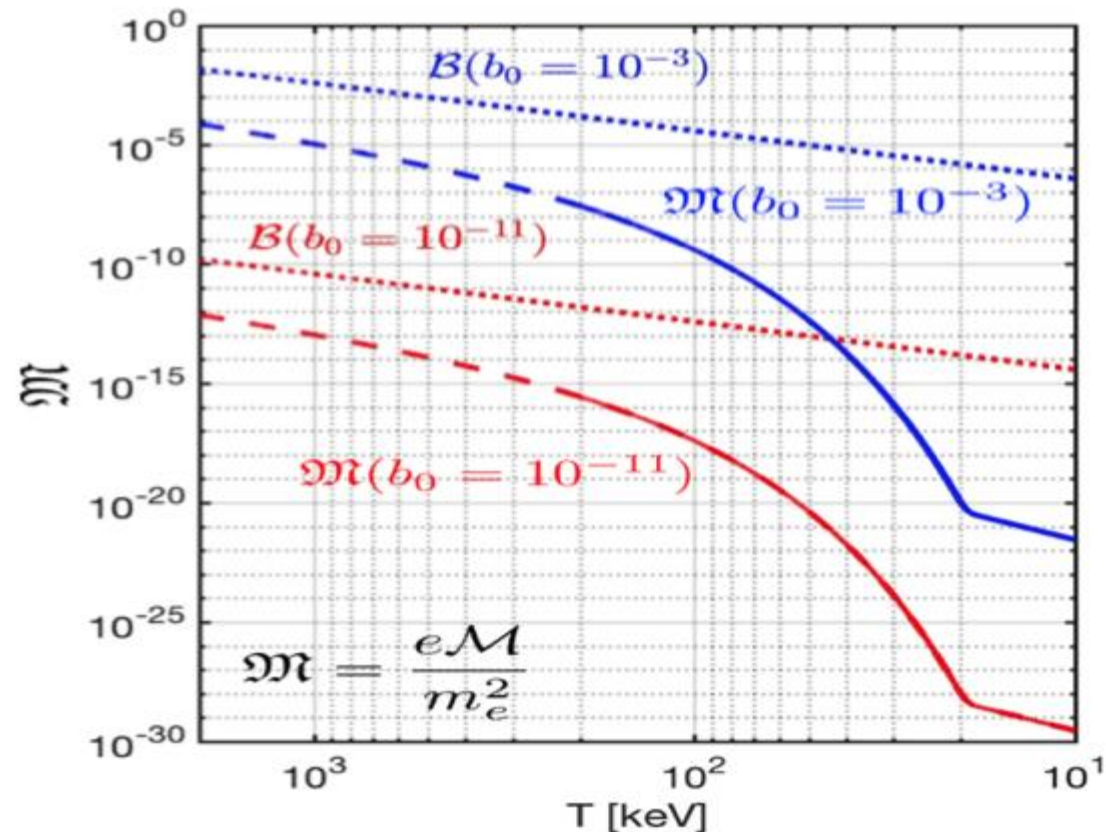
$$x_i = \tilde{m}_i / T \quad \tilde{m}_{\pm}^2 = m_e^2 + eB \left(1 \mp \frac{g}{2} \right)$$

First approximation:

$$\frac{\eta_s}{T} \ll 1$$

J.Rafelski, J.Birrell, A.Steinmetz and C.T.Yang, "A short survey of matter-antimatter evolution in the primordial universe," Universe 9, 309 (2023)

We can explore the electron-positron magnetization process in plasma via the partition function.



Magnetization

$$\mathcal{M} = \left(\frac{T}{V} \frac{\partial \ln \mathcal{Z}_{e^+e^-}}{\partial B} \right)$$

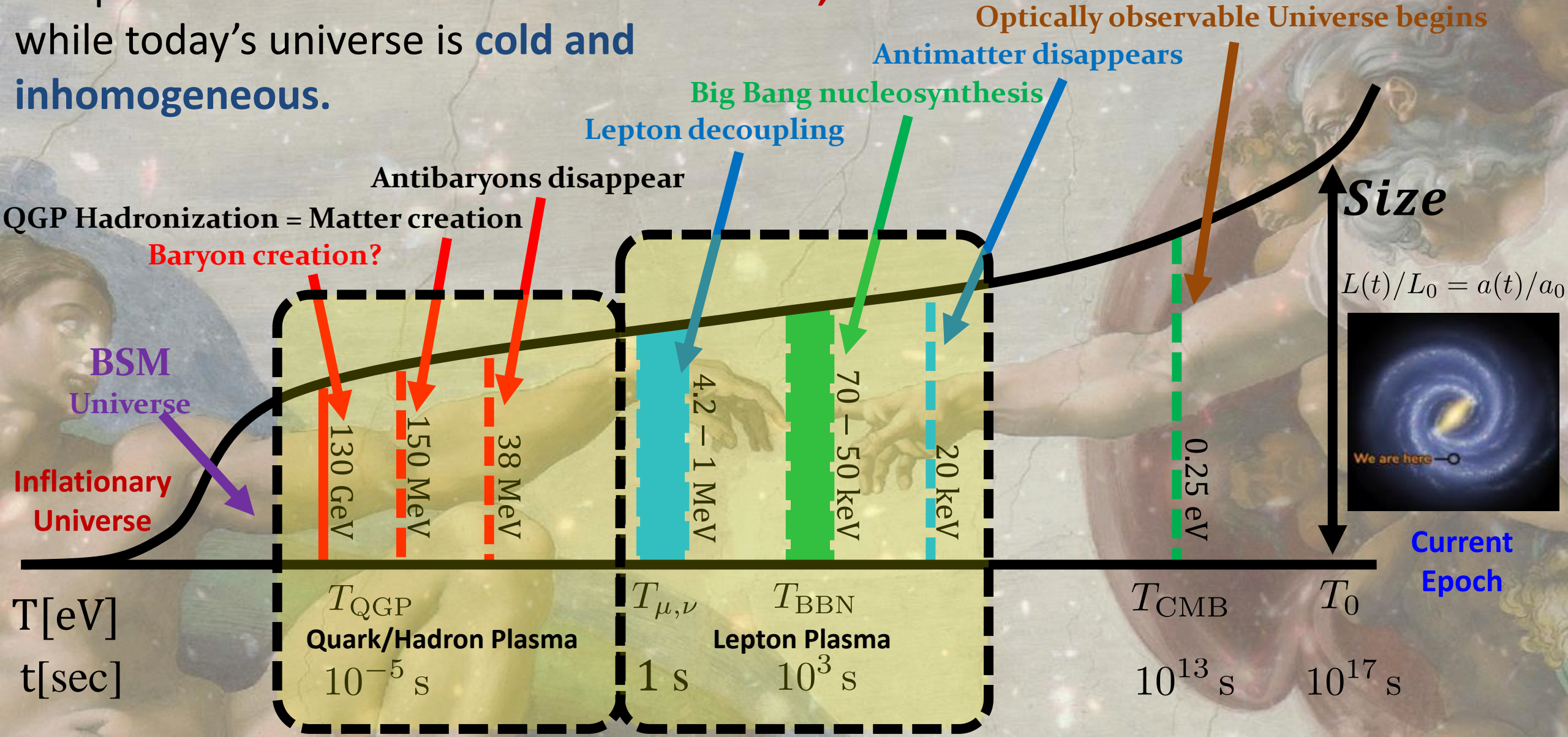
- The e⁻e⁺-plasma has paramagnetic properties when subjected to an external field
- The magnetization never exceeds the external field under the parameters considered.
- The research of nonzero spin chemical potential is continue by Andrew Steinmetz's Phd thesis and paper. **A. Steinmetz, C.T. Yang and J. Rafelski, "Matter-antimatter origin of cosmic magnetism," [arXiv:2308.14818 [hep-ph]], Phys. Rev.D 108, 12, 123522(2023)**

The image is a reproduction of Michelangelo's famous fresco, "The Creation of Adam." It depicts Adam on the left, reclining on a rock, and God on the right, reclining on a rock. The two figures are positioned such that their hands are just inches apart, creating a sense of tension and divine spark. The background is a light, textured surface with some visible cracks and a subtle pattern of small, glowing particles. The text "Take Home Points" is centered over the middle of the image.

Take Home Points

Major events in the Primordial Universe

The primordial universe was **hot and dense**, while today's universe is **cold and inhomogeneous**.



Our recent work in cosmology

- ❑ **This review submitted to EPJ ST**
- ❑ Grayson, C., et al. "Self-consistent strong screening applied to thermonuclear reactions." arXiv preprint arXiv:2406.13055 (2024).
- ❑ Birrell, J., et al. "Fermi-Dirac Integrals in Degenerate Regimes: Novel Asymptotic Expansion." Int.J.Theor.Phys. 63.7 (2024): 163.
- ❑ Rafelski, J., et. al. "Dynamic fermion flavor mixing through transition dipole moments." Int.J.Mod.Phys.A 38.31 (2023): 2350163.
- ❑ Steinmetz, A., C. T. Yang, and J. Rafelski. "Matter-antimatter origin of cosmic magnetism." Physical Review D 108.12 (2023): 123522.
- ❑ Grayson, C., et al. "Electron–positron plasma in BBN: Damped-dynamic screening." Annals of Physics 458 (2023): 169453.
- ❑ Rafelski, J., et al. "A short survey of matter-antimatter evolution in the primordial universe." Universe 9.7 (2023): 309.
- ❑ Grayson, C., et al. "Dynamic magnetic response of the quark-gluon plasma to electromagnetic fields." Physical Review D 106.1 (2022): 014011.
- ❑ Yang, C. T., and J. Rafelski. "Cosmological strangeness abundance." Physics Letters B 827 (2022): 136944.
- ❑ Rafelski, J., and C. T. Yang. "The muon abundance in the primordial Universe." Acta Phys. Polon. B 52.arXiv: 2103.07812 (2021): 277.

PHYSICAL REVIEW D **108**, 123522 (2023)

Matter-antimatter origin of cosmic magnetism

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 (Received 30 August 2023; accepted 15 November 2023; published 11 December 2023)

Annals of Physics 458 (2023) 169453

Electron–positron plasma in BBN: Damped-dynamic screening



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Review

A Short Survey of Matter-Antimatter Evolution in the Primordial Universe

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Prof. Johann Rafelski



Prof. Andrew Steinmetz



Prof. Jeremiah Birrell



Dr. Christopher Grayson



Dr. Cheng Tao Yang



Shelbi J. Foster

Highlights and Thanks

- The universe's evolution contains distinct eras of different particles forming the plasma.
- Standard particle model plasma predict to contain long lasting chemical nonequilibrium.
- Antibaryons disappear at 38.2 MeV; long lasting strangeness and muons to 4.2 MeV.
- Progressive decoupling and free streaming neutrino at a few MeV.
- Positron disappear at 20.3 keV, electron-positron plasma as a source of magnetization in study.

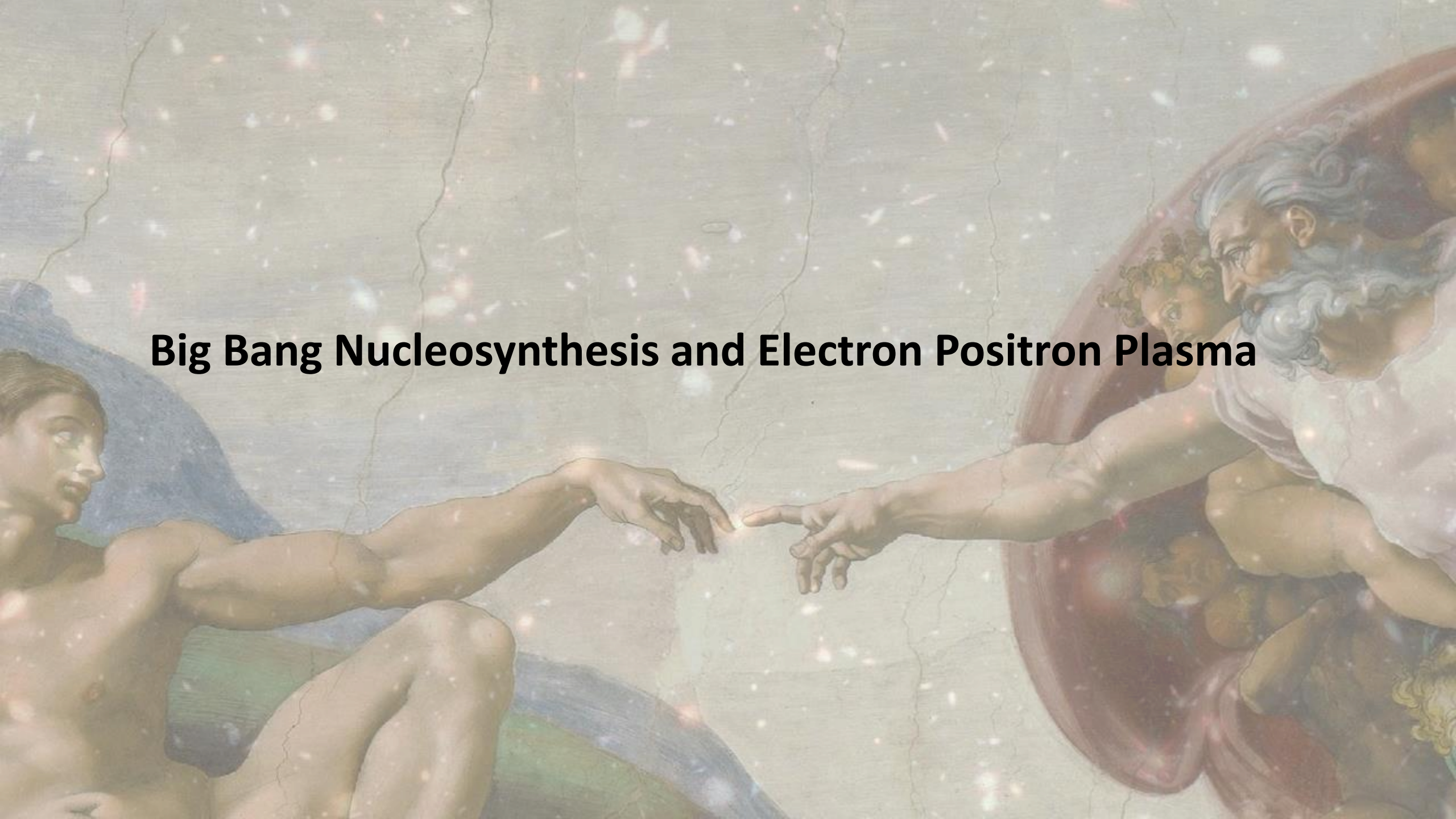
Thank you to Elder Prof. Anthony Thomas and Springer Nature for sponsorship.



The image is a reproduction of Michelangelo's famous fresco, "The Creation of Adam." It depicts Adam on the left, reclining on a rock, and God on the right, reclining on a rock. The two figures are positioned such that their hands are just inches apart, creating a sense of tension and divine spark. The background is a light, textured surface with some visible cracks and a subtle pattern of small, glowing particles. The text "Supplementary Slides" is centered over the image in a bold, black, sans-serif font.

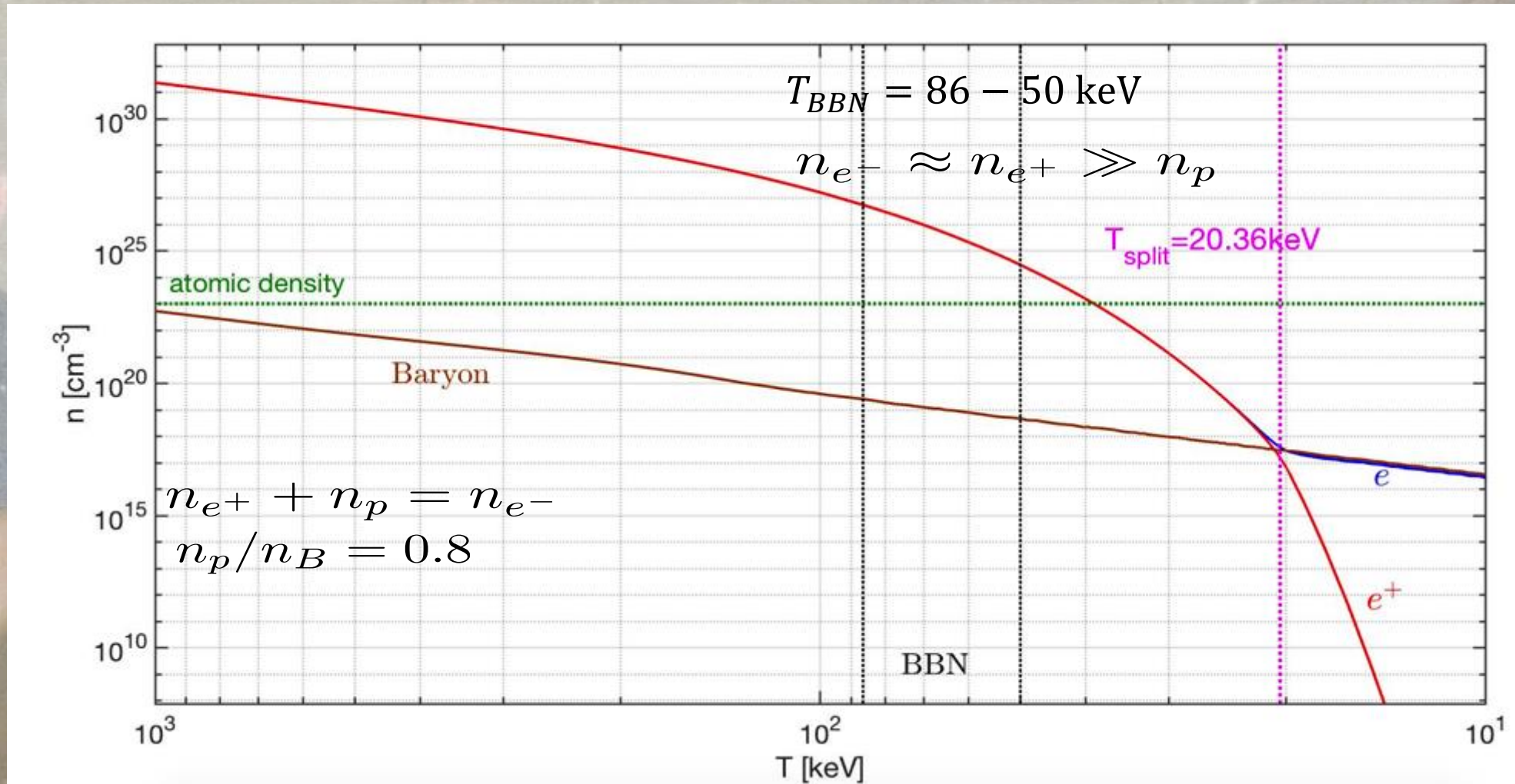
Supplementary Slides

Big Bang Nucleosynthesis and Electron Positron Plasma



Big Bang Nucleosynthesis in rich Electron/Positron Plasma

We will show that when the temperature is $T = 86$ keV, we have an abundance of electrons and positrons in cosmic plasma with $n_{e^\pm}/n_B = 10^7$



BBN happens within the rich electron-positron plasma and **plasma effects need to be accounted for in the precision study of the final abundances of light elements produced in BBN.**

Electron Chemical Potential and Charge Neutrality

The dense electron/positron plasma in the primordial universe is neutralized by protons. The proton abundance is derived by using entropy conservation. **The presence of rich electron positron plasma can last until $T=20.3$ keV,** and the plasma effect needs to be accounted in study of BBN.

Charge neutrality :

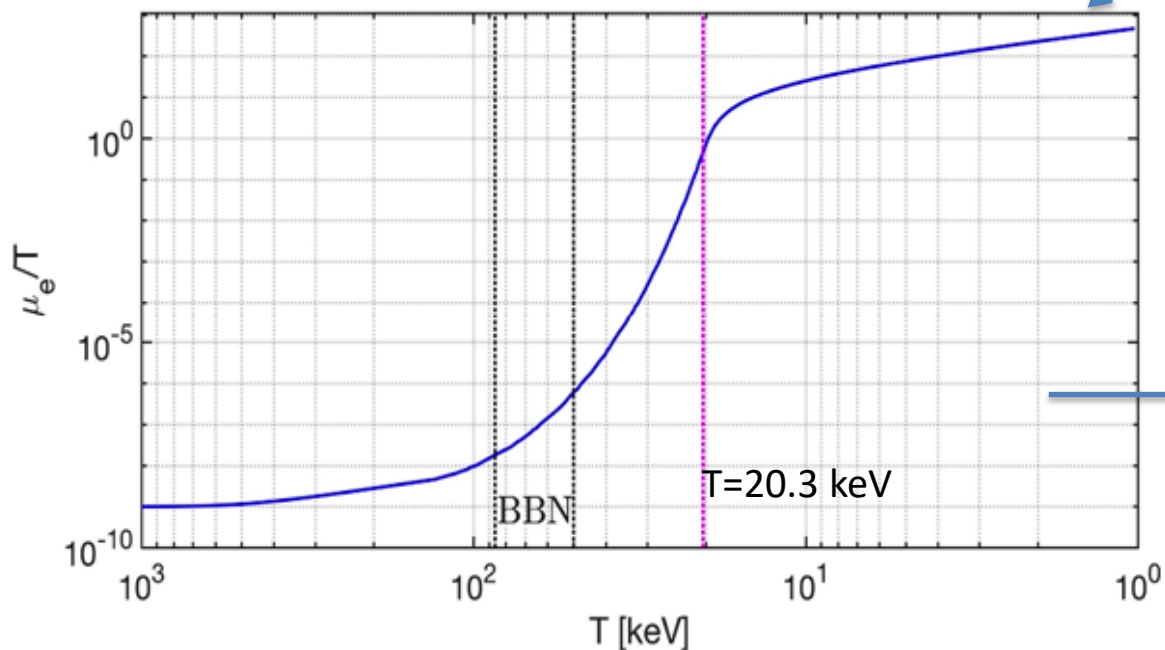
$$n_e - n_{\bar{e}} = \frac{n_p}{n_B} \left(\frac{n_B}{s_{\gamma, e, \bar{e}}} \right) s_{\gamma, e, \bar{e}}$$

Constant baryon-to-entropy ratio

Observation

$$(n_B/n_\gamma)_{t_0} = 6.05 \times 10^{-10} \quad n_p/n_B = 0.87$$

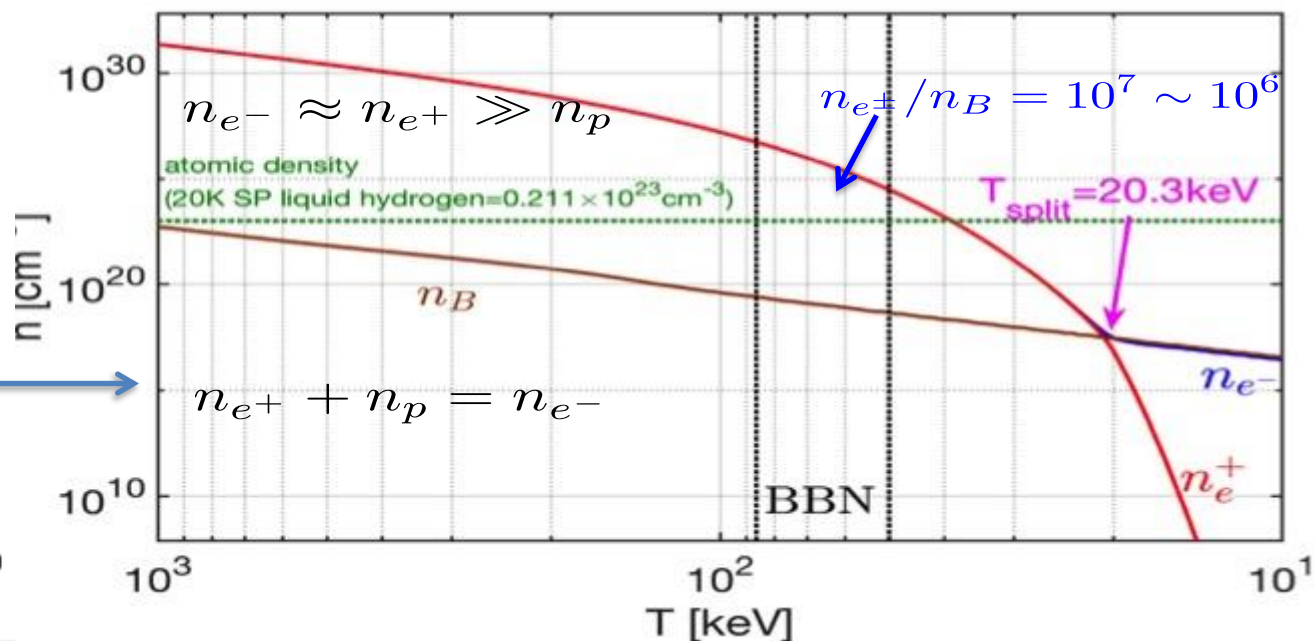
$$T_{\text{BBN}} = 86 \sim 50 \text{ keV}$$



The number density of electrons over positrons

$$n_e - n_{\bar{e}} = \frac{g_e}{2\pi^2} \left[\int_0^\infty \frac{p^2 dp}{\exp((E - \mu_e)/T_\gamma) + 1} - \int_0^\infty \frac{p^2 dp}{\exp((E + \mu_e)/T_\gamma) + 1} \right]$$

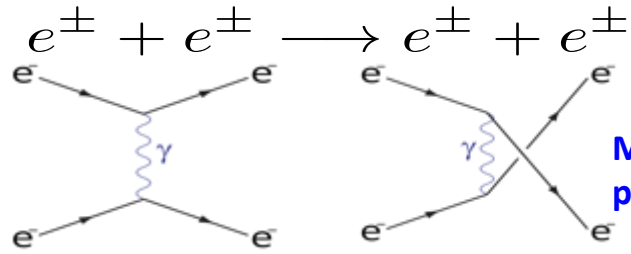
$$T_{\text{BBN}} = 86 \sim 50 \text{ keV}$$



Microscopic evaluation of collision damping in e⁻e⁺-plasma

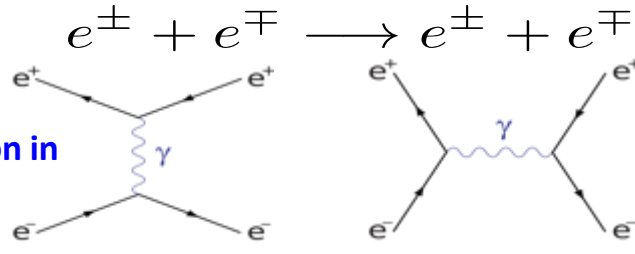
In electron-positron plasma the major reactions between photons and electron/positron pairs are given by

Møller scattering:

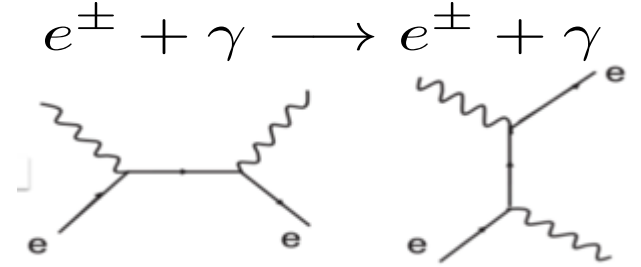


Massive photon in plasma

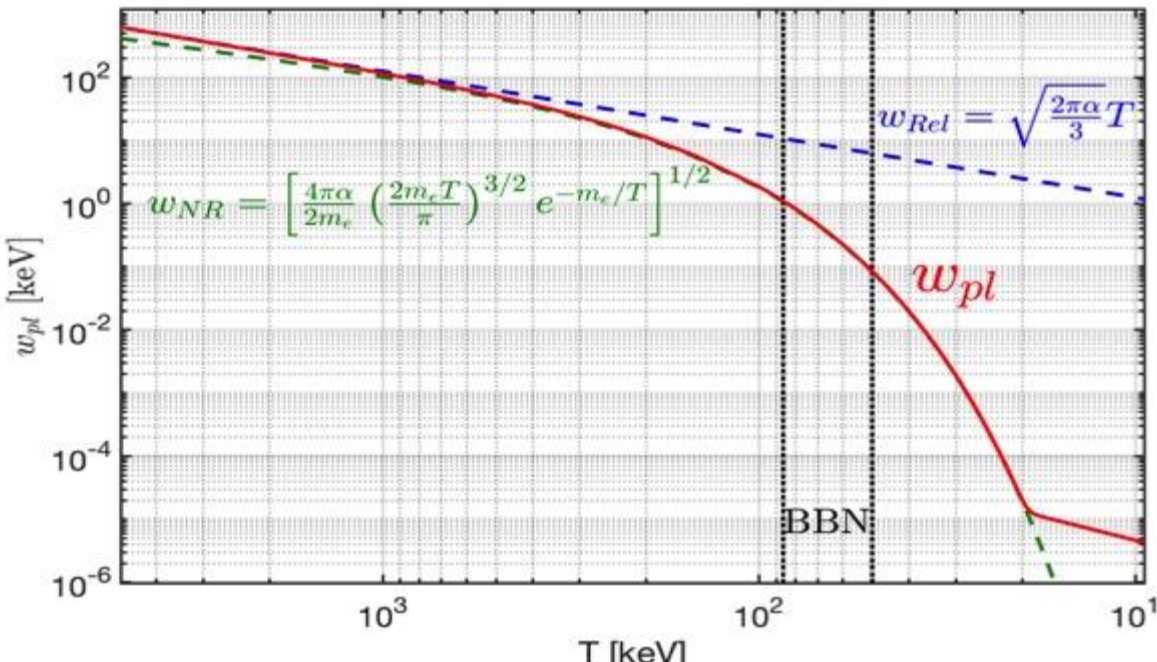
Bhabha scattering:



Inverse Compton scattering:



A photon propagates through a plasma of electrons and positrons, its propagation is influenced by the interactions with the medium and generate effective mass for the photon in plasma.



For the BBN temperature $50 < T_{\text{BBN}} < 86$ keV, we can consider the Boltzmann approximation for the nonrelativistic electron/positron plasma:

$$m_\gamma^2 = w_{pl}^2 = \frac{4\pi\alpha}{2m_e} \left(\frac{2m_e T}{\pi} \right)^{3/2} e^{-m_e/T}$$

$$T_{\text{BBN}} = 86 \sim 50 \text{ keV}$$

$$m_\gamma = w_{pl} = 1 \sim 0.1 \text{ keV}$$

Photons still remain relativistic because $w_{pl}/T_{\text{BBN}} \ll 1$.

Damping rate in electron-positron plasma

To calculate the damping rate κ in plasma, we calculate the reaction rate per volume in two-body reaction in the Boltzmann approximation first:

$$R_{12 \rightarrow 34} = \langle \sigma v \rangle_T n_1 n_2 = \frac{g_1 g_2}{32\pi^4} \frac{T}{1 + I_{12}} \int_{s_{th}}^{\infty} ds \sigma(s) \frac{[s - (m_1 + m_2)^2][s - (m_1 - m_2)^2]}{\sqrt{s}} K_1(\sqrt{s}/T),$$

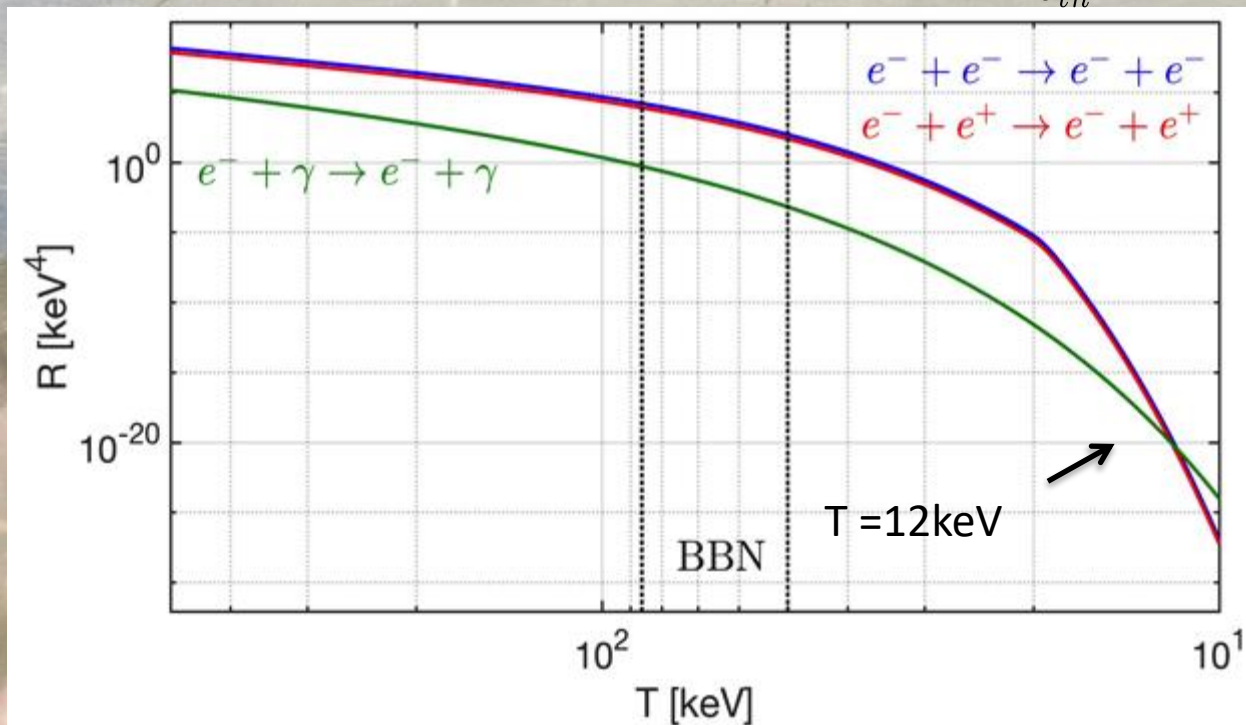


Jean Letessier, Johann Rafelski, "Hadrons and Quark-Gluon Plasma"

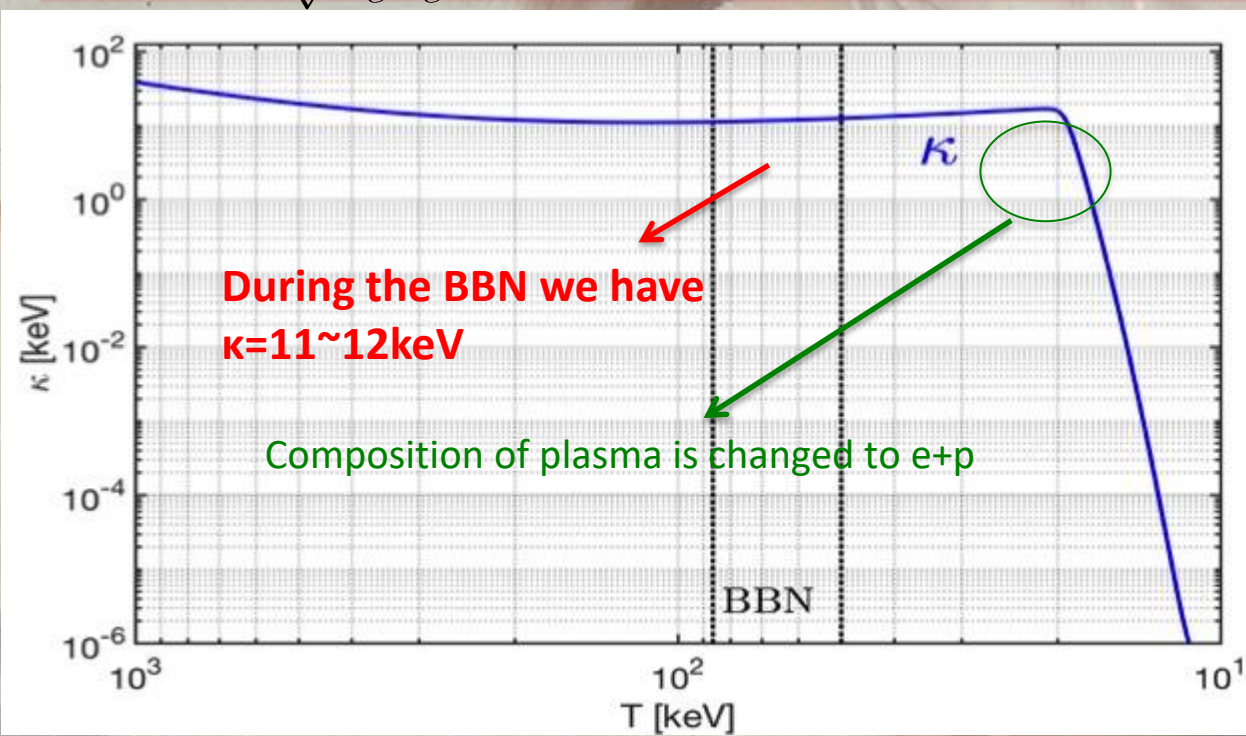
For $T > 12 \text{ keV}$, the dominate reactions in plasma are Møller and Bhabha scatterings

The electron(positron) damping rates can be defined as

$$\kappa = \frac{R_{e\bar{e} \rightarrow e\bar{e}} + R_{ee \rightarrow ee}}{\sqrt{n_e n_{\bar{e}}}} \quad \sqrt{n_e n_{\bar{e}}} = \frac{g_e}{2\pi^3} T^3 \left(\frac{m_e}{T}\right)^2 K_2(m_e/T)$$



For comprehensive discussion and the application of the damped e^-e^+ -plasma and dynamic screening during BBN see paper: **C. Grayson, C.T. Yang, M. Formanek and J. Rafelski, "Electron-positron plasma in BBN: Damped-dynamic screening," Annals Phys. 458, 169453 (2023)**



Improving damping rate in e^-e^+ -plasma: self-consistence approach

The photon mass is one of important parameters in the calculation of the relaxation rate for Møller and Bhabha scattering in electron positron plasma.

Getting BBN right requires good understanding of dense e^\pm plasma!

M. Formanek, C. Grayson, J. Rafelski and B. Müller, "Current-conserving relativistic linear response for collisional plasmas," *Annals Phys.* 434, 168605 (2021)

Considering the linear response theory, the dispersion relation for the photon in nonrelativistic e^\pm plasma is given by

$$w^2 = |k|^2 + \frac{w}{w + i\kappa} w_{pl}^2 \xrightarrow{\text{Solving the dispersion relation with } k=0} w_{\pm} = -i\frac{\kappa}{2} \pm \sqrt{w_{pl}^2 - \frac{\kappa^2}{4}}$$

The effective plasma frequency in damped plasma

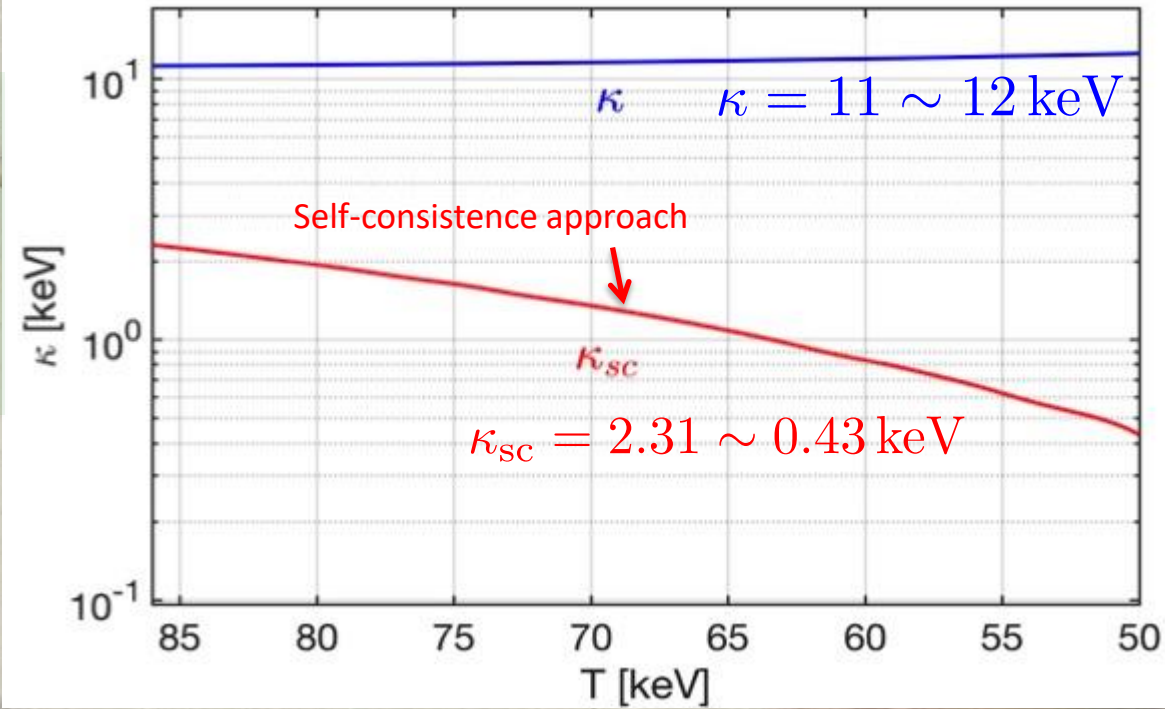
The effective photon mass in damped plasma is a function of the scattering rate.

$$m_{\gamma}(\kappa, w_{pl})$$

we need to solve the self-consistent equation for scattering rate:

$$\kappa \left[\frac{g_e}{2\pi^3} T^3 \left(\frac{m_e}{T} \right)^2 K_2(m_e/T) \right] = \frac{g_e g_e}{32\pi^4} T \int_{4m_e^2}^{\infty} ds \frac{s(s - 4m_e^2)}{\sqrt{s}} K_1(\sqrt{s}/T) \left[\sigma_{e^\pm e^\pm}(s, w_{pl}, \kappa) + \sigma_{e^\pm e^\mp}(s, w_{pl}, \kappa) \right]$$

To solve the self-consistent equation we need to know the transition amplitude for Møller and Bhabha scattering with imaginary photon mass which requires detail study.



Electron-positron magnetization in primordial Universe

The Universe today filled with magnetic fields at various scales and strengths both within galaxies and in deep extra-galactic space far and away from matter sources

Observation: Extra-galactic magnetic fields (EGMF)

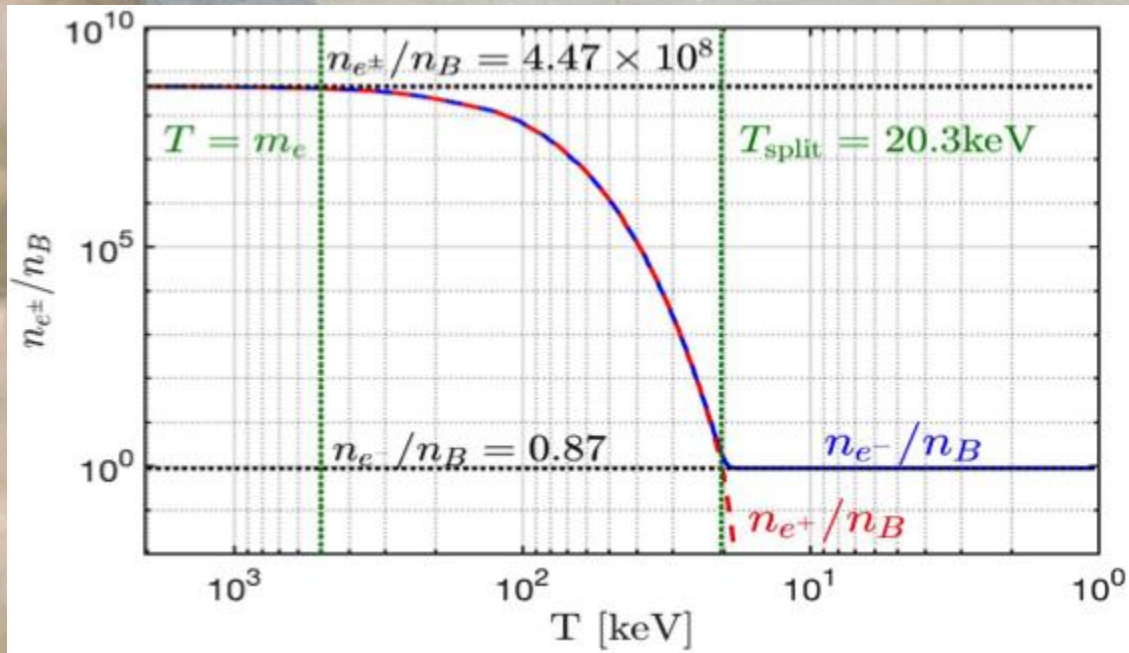
$$10^{-8} \text{ G} > \mathcal{B}_{\text{EGFM}} > 10^{-16} \text{ G}$$



- Electrons and positrons have **the largest magnetic moments** in nature.



This very dense electron-positron plasma could be the origin of cosmic magnetism.



The partition of the electron-positron plasma in a uniform magnetic field B pointing along the z -axis

$$\ln \mathcal{Z}_{e^+e^-} = \frac{eBV}{(2\pi)^2} \sum_{\sigma}^{\pm} \sum_{s}^{\pm} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} dp_z \left[\ln \left(1 + \lambda_{\sigma} \xi_s e^{-E_n^s/T} \right) \right]$$

$$E_n^{\pm} = \sqrt{p_z^2 + \tilde{m}_{\pm}^2 + 2eBn} \quad \tilde{m}_{\pm}^2 = m_e^2 + eB \left(1 \mp \frac{g}{2} \right)$$

Particle chemical potential

$$\lambda_{\pm} = \exp(\pm \mu_e/T)$$

Spin chemical potential

$$\xi_{\pm} = \exp(\pm \eta_s/T)$$

We study the magnetization process within dense electron-positron plasma via fundamental quantum statistical analysis:

Pshirkov, M. S., P. G. Tinyakov, and F. R. Urban. "New limits on extragalactic magnetic fields from rotation measures". *Phys. Rev. Lett.*, 116(19), p. 191302(2016)

Jedamzik, K. and A. Saveliev, "Stringent limit on primordial magnetic fields from the cosmic microwave background radiation". *Physical review letters*, 123(2), p. 021301. (2019).

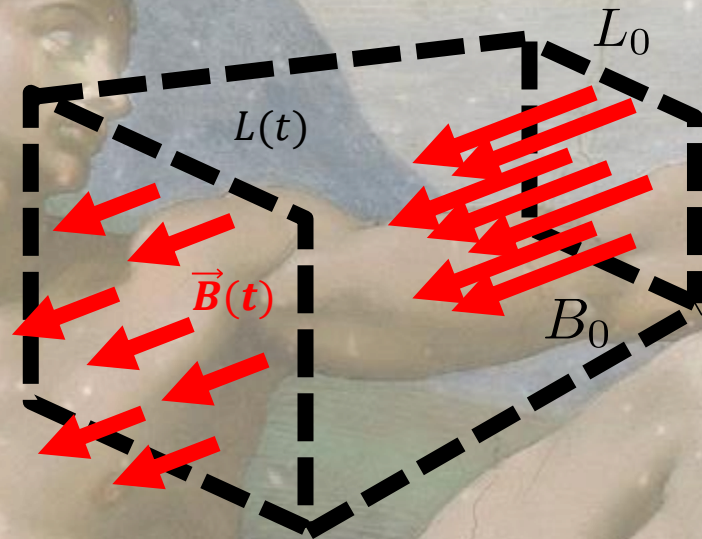
Example: How magnetism is function of temperature

Magnetic flux is conserved over a comoving surface

$$L(t)^2 = L_0^2 a(t)^2 / a(t_0)^2$$

The magnetic field then dilutes over time as

$$B(t)^2 = B_0^2 a(t_0)^2 / a(t)^2$$



Magnetic flux:

$$\Phi_B = B(t)L^2(t) = B_0L_0^2$$

Constant relic magnetic scale

The temperature also decreases over cosmic expansion as

$$T(t) = T_0 a(t_0) / a(t)$$

Magnetic fields present in cosmic voids would be “uncontaminated” primordial relics.

This lets us define a conserved cosmic “magnetic scale” for charged particles.

$$b_0 \equiv eB(t)/T(t)^2 = eB_0/T_0^2 = \text{const}$$

In natural units:

$$\hbar = c = k_B = 1$$

Contemporary B-fields

$$10^{-12} \text{T} > B_0 > 10^{-20} \text{T}$$

Upper: Faraday rotation of radio AGN
Lower: Spectra of “blazar” AGN

Contemporary temperature

$$T_0 = 2.7 \text{ K} = 2.3 \times 10^{-4} \text{ eV}$$

As determined from the Cosmic Microwave Background (CMB)

$$10^{-3} > b_0 > 10^{-11}$$

Thus b_0 controls the strength of the magnetization of the primordial electron-positron plasma.

Note: B-field decreases with decreasing temperature.

The upper and lower bounds on inter-galactic magnetic fields (IGMF) coherent over Mpc scales is shown.

Chemical equilibrium: Composition of hadronic matter universe

We show that the Universe in the range $T > 20$ MeV is rich in physics phenomena involving strange mesons, (anti)baryons including (anti)hyperon abundances.

For a given chemical potential the number density in Boltzmann limit is:

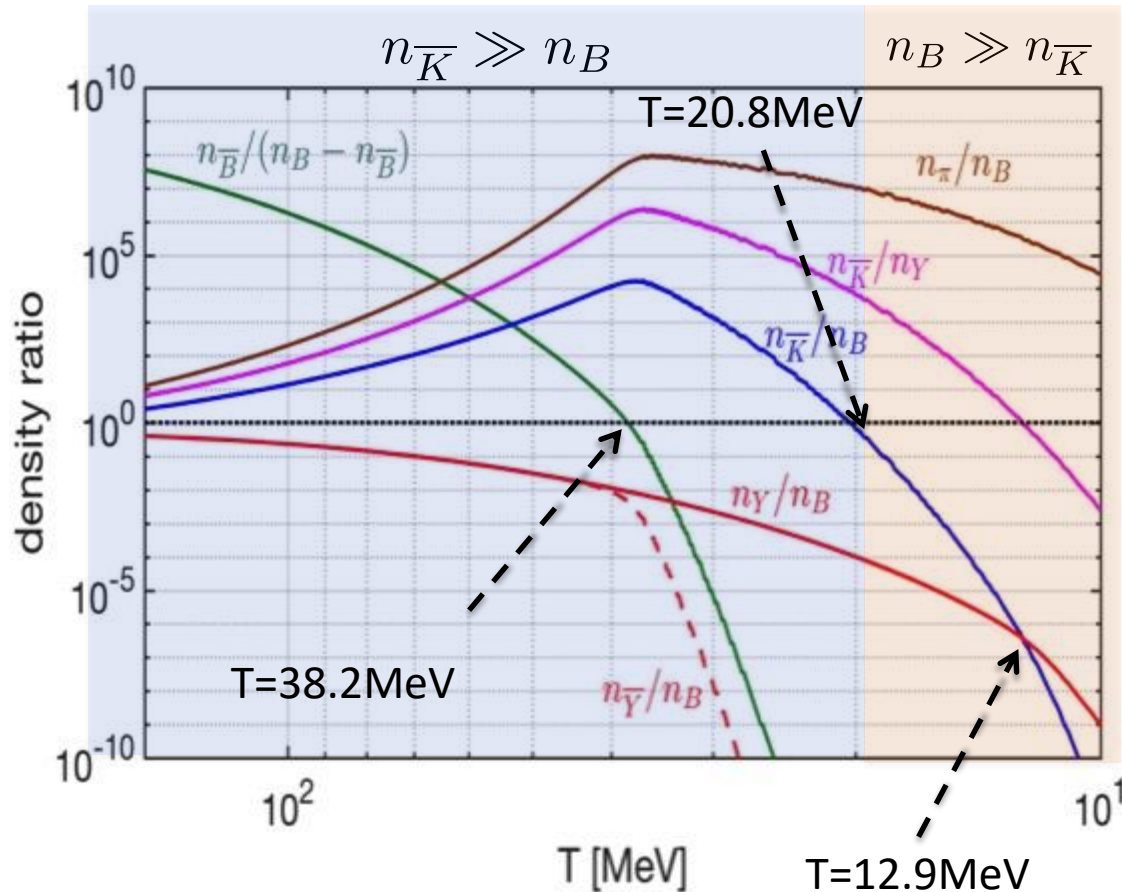
$$n_K = \frac{T^3}{2\pi^2} (\lambda_s \lambda_q^{-1}) \sum_{K_i} g_{K_i} \left(\frac{m_{K_i}}{T}\right)^2 K_2(m_{K_i}/T) \quad n_Y = \frac{T^3}{2\pi^2} (\lambda_q^2 \lambda_s) \sum_{Y_i} g_{Y_i} \left(\frac{m_{Y_i}}{T}\right) K_2(m_{Y_i}/T)$$

$$\mu_s = T \ln \lambda_s \quad \mu_B = 3T \ln \lambda_q$$

Requirement: Charge neutrality

Prescribe value of entropy-per-baryon $S/B = 3.5 \times 10^{10}$

Assume: Net lepton number equals net baryon number



For $T=38.2$ MeV antibaryons disappear from the Universe inventory
 $n_{\bar{B}}/(n_B - n_{\bar{B}}) = 1 \rightarrow T = 38.2$ MeV

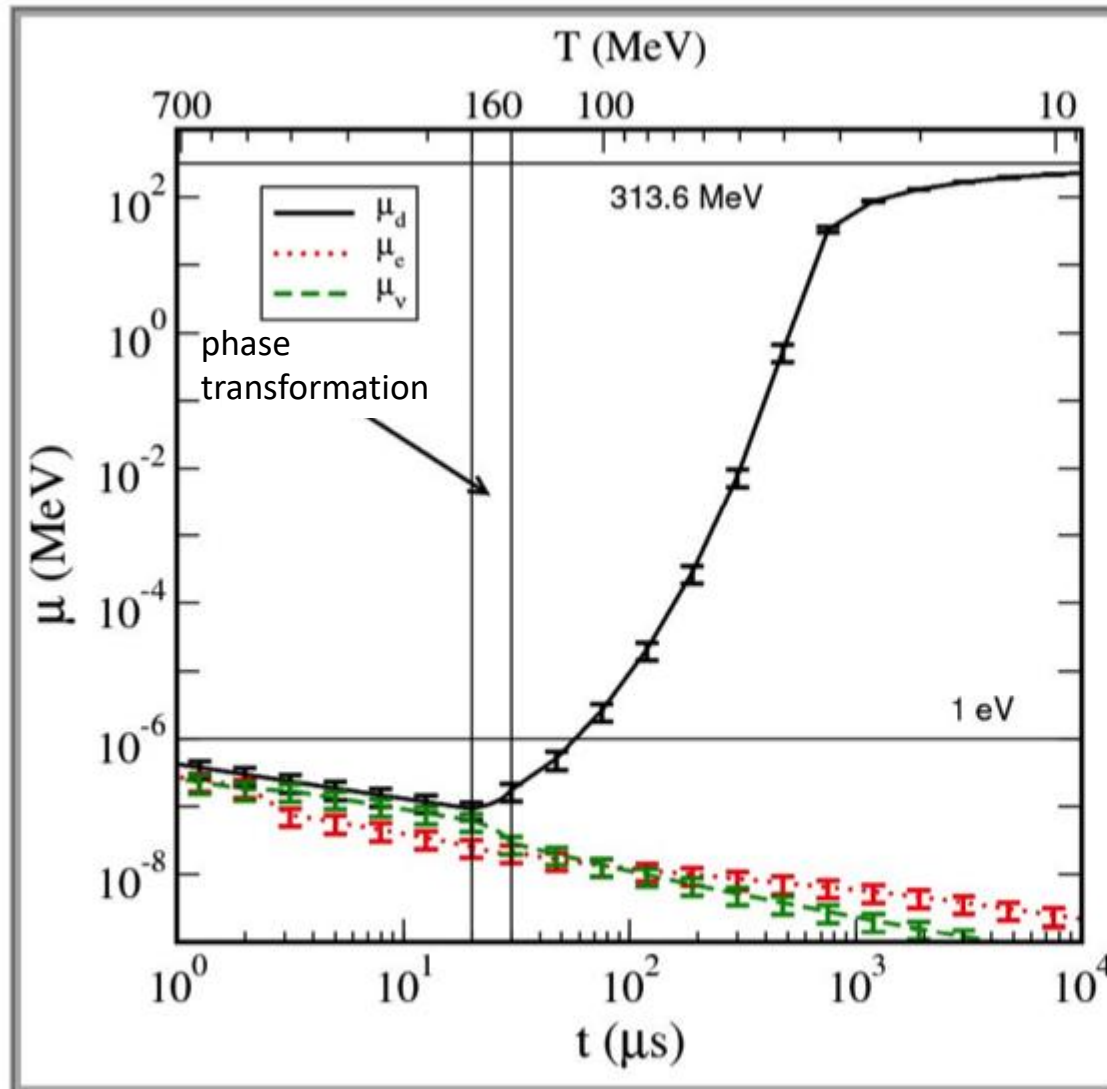
In qualitative agreement with Kolb and Turner $T=40$ MeV
 Kolb, E.W.; Turner, M.S. *The early universe*; CRC Press, 1990

For $T > 20.8$ MeV, strangeness is dominantly in meson

For 20.8 MeV $> T > 12.9$ MeV, strangeness present in hyperons and anti-strangeness in kaon keeping strangeness symmetry $\bar{K} + N \rightarrow \Lambda + \pi$

For 12.9 MeV $> T$ strangeness is dominantly present in hyperons

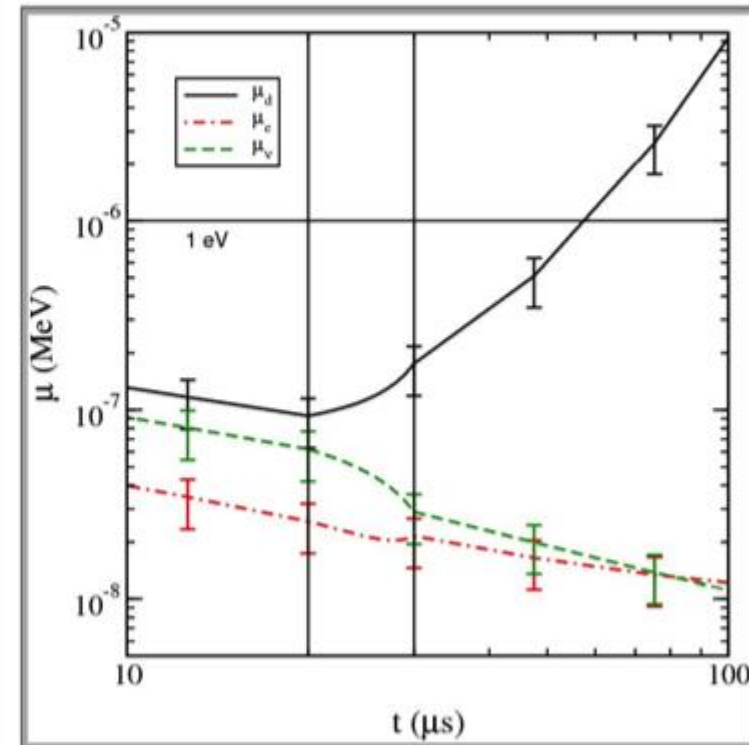
Chemical potential created by baryon asymmetry



*2002 input data:
Changes in 2022 data nearly within
the error bars and hardly visible*

Minimum:

$$\mu_B = 0.33^{+0.11}_{-0.08} \text{ eV}$$

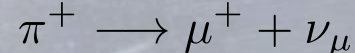
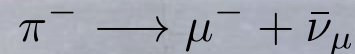
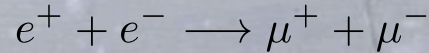
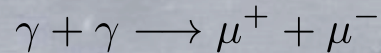


$\Rightarrow \mu_B$ defines remainder of matter after annihilation

Lepton Universe: Muon persistence temperature

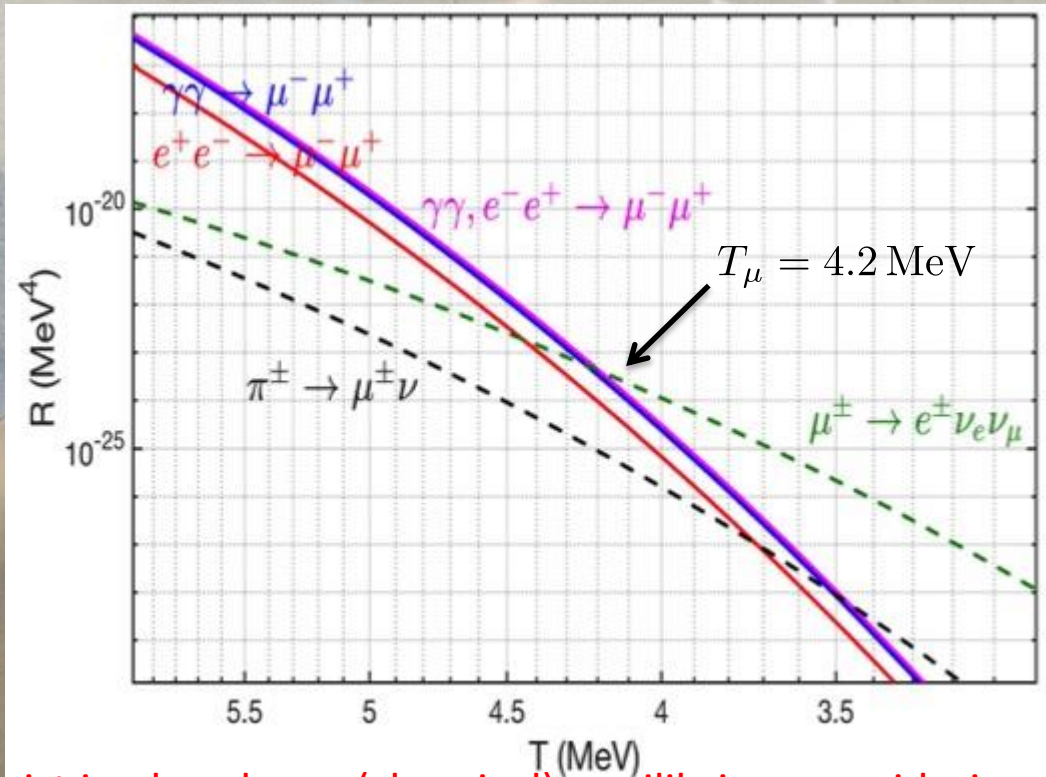
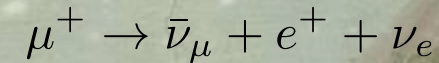
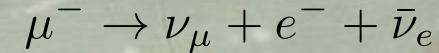
We establish the range of temperature in which production processes exceed in speed the decay process that keep muons in abundance (chemical) equilibrium

Muon Production



$$\tau_\pi = 2.6033 \times 10^{-8} \text{ sec}$$

Muon Decay $\tau_\mu = 2.197 \times 10^{-6} \text{ sec}$



$$R_{a\bar{a} \rightarrow b\bar{b}} = \frac{g_a g_{\bar{a}}}{1 + I} \frac{T}{32\pi^4} \int_{s_{th}}^{\infty} ds \frac{s(s - 4m_a^2)}{\sqrt{s}} \sigma_{a\bar{a} \rightarrow b\bar{b}} K_1(\sqrt{s}/T)$$

$$R_c = \frac{g_c}{2\pi^2} \left(\frac{T^3}{\tau_c}\right) \left(\frac{m_c}{T}\right)^2 K_1(m_c/T)$$

Vacuum decay rate applies because the $T > m_e$, decay blocking is small.

As the temperature decreases in the expanding Universe, the muon abundance disappears as soon as any decay rate is faster than the fastest production rate.

Specifically after the Universe cools below the temperature $T = 4.2 \text{ MeV}$, the dominant reaction is the muon decay.

Muons persist in abundance (chemical) equilibrium considering electromagnetic and weak interaction processes until $T = 4.2 \text{ MeV}$.

Electroweak 2-2 processes in progress.



Lepton asymmetry: $B \neq L$ Effective number of neutrinos and μ_ν

We study in a quantitative manner the lepton asymmetry L in the Universe and its impact on Universe expansion. **To be specific we explore how the large cosmological lepton asymmetry relates to the effective number of neutrinos.**

$$N_\nu^{\text{eff}} \equiv \frac{\rho_\nu^{\text{tot}}(\mu_\nu)}{\frac{7\pi^2}{120} \left(\frac{4}{11}\right)^{4/3} T_\gamma^4} \longleftrightarrow \mu_\nu \longleftrightarrow L \equiv \frac{[N_L(\mu_\nu) - N_{\bar{L}}(\mu_\nu)]}{N_\gamma}$$

Free-streaming distribution: Birrell, J., C.-T. Yang, P. Chen, and JR (2014a). Phys.Rev. D, 89, p. 023008.

$$f_\nu = \frac{1}{\exp\left(\sqrt{\frac{E^2 - m_\nu^2}{T_\nu^2} + \frac{m_\nu^2}{T_f^2}} - \sigma \frac{\mu_\nu}{T_f}\right) + 1}$$

$$T_\nu \equiv \frac{a(t_f)}{a(t)} T_f \quad T_f = 2 \text{ MeV} \quad \sigma = \pm 1$$

Effective number of neutrino

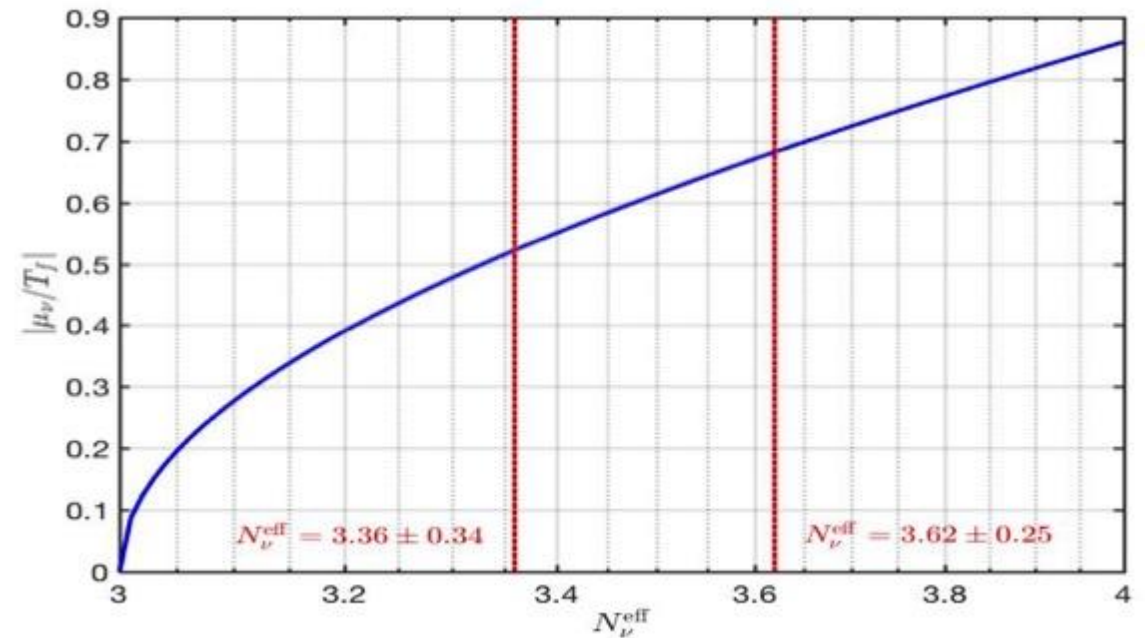
$$N_\nu^{\text{eff}} = 3 \left[1 + \frac{30}{7\pi^2} \left(\frac{\mu_\nu}{T_f}\right)^2 + \frac{15}{7\pi^4} \left(\frac{\mu_\nu}{T_f}\right)^4 \right]$$

The SM value of the effective number of neutrinos can be obtained when chemical potential is small:

$$N_\mu^{\text{eff}} = 3 \longleftrightarrow \mu_\nu \ll T_f$$

To interpret the literature values of effective number of neutrino, we require

$$0.52 \leq \mu_\nu / T_f \leq 0.69 \quad \text{neutrino-antineutrino asymmetry freezeout}$$



Effective number of neutrinos and lepton number

The density ratio between lepton-asymmetry and baryon-asymmetry at neutrino freeze-out can be written as

$$\frac{l_f}{b_f} \equiv \left(\frac{n_e - n_{\bar{e}}}{n_B} \right)_f + \sum_{i=e,\mu,\tau} \left(\frac{n_{\nu_i} - n_{\bar{\nu}_i}}{n_B} \right)_f \quad (n_\nu - n_{\bar{\nu}})_f = \frac{g_\nu}{6\pi^2} T_f^3 \left[\pi^2 \left(\frac{\mu_\nu}{T_f} \right) + \left(\frac{\mu_\nu}{T_f} \right)^3 \right]$$

Charge neutrality:
It is small compared to the 2nd term

The subscript f indicate the quantities at the neutrino freezeout temperature

Lepton-baryon number ratio

$$\frac{L}{B} \approx \sum_{i=e,\mu,\tau} \left(\frac{n_\nu - n_{\bar{\nu}}}{\sigma} \right)_f \left(\frac{\sigma}{n_B} \right)_f = \frac{45}{4\pi^4} \frac{\pi^2 (\mu_\nu/T_f) + (\mu_\nu/T_f)^3}{10.75 + \frac{45}{4\pi^2} (\mu_\nu/T_f)^2} \left(\frac{\sigma}{n_B} \right)_{t_0}$$

$$B = \frac{[N_B - N_{\bar{B}}]}{N_\gamma} \quad L \equiv \frac{[N_L - N_{\bar{L}}]}{N_\gamma}$$

$$\sigma_f = \frac{2\pi^2}{45} \left[10.75 + \frac{45}{4\pi^2} \left(\frac{\mu_\nu}{T_f} \right)^2 \right] T_f^3$$

Effective number of neutrino

$$N_\nu^{\text{eff}} = 3 \left[1 + \frac{30}{7\pi^2} \left(\frac{\mu_\nu}{T_f} \right)^2 + \frac{15}{7\pi^4} \left(\frac{\mu_\nu}{T_f} \right)^4 \right]$$

Our point of view:

Instead of $B \simeq |L|$, we found that $0.4 \leq |L| \leq 0.52$ and $B \simeq 1.33 \times 10^{-9} |L|$ reconciles the CMB and current epoch results for the Hubble expansion parameter

Yang, C. T., J. Birrell, and JR. "Lepton Number and Expansion of the Universe." arXiv preprint arXiv:1812.05157 (2018).

