

Eckle *et al.* Reply: In the preceding Comment [1], Affleck and Simon argue that the approach adopted in our Letter [2] is not appropriate for calculating the persistent current in a mesoscopic ring with a sidebranch quantum dot, when tuned to a Kondo resonance. Specifically, they suggest that our assumption of a linear dispersion relation is the reason why we obtain a result in contradiction to theirs [3,4]. In response to this, we point out that the use of a linear dispersion relation for calculating persistent currents is by now well established and has been extensively exploited for the case of studying, e.g., parity effects [5] or effects from disordering potentials [6] in 1D rings. Our definition of the persistent current in terms of *excess numbers* [Eq. (7) in [2]] is conceptually the same as that in [5] where the persistent current is defined via a topological number describing the difference between the number of right- and left-moving particles. In our Letter [2], we argue that since the Kondo resonance of the ring and the sidebranch dot is a Fermi-level property, the use of a linear dispersion and, *a fortiori*, the definition in Eq. (7) in [2], should remain valid also in the presence of the dot. We admit that this is an assumption that needs to be carefully examined. Although the curvature of the “true” dispersion close to the Fermi level is known to generate only irrelevant operators in 1D (in renormalization-group sense) [7], it is still conceivable that these operators could “tie together” the charge and spin sectors of the integrable theory in such a way as to modify the persistent current. We are presently investigating this possibility [8].

Considering the approach advocated by Affleck and Simon [3,4], we note that it rests on the assumption that a persistent current can be calculated as if it were a transport current, and hence is completely determined by the transmission amplitude of an electron at the Fermi level. This is a nontrivial assertion, which has so far been demonstrated convincingly only for the case of a 1D ring with potential scatterers [9]. However, in an interacting 1D system with spin-charge separation, a finite persistent current will result if *collective charge excitations* can encircle the ring, even if an electron, which carries both *charge and spin*, cannot. In our Letter [2], we argued that this is precisely the case for Kondo scattering in a 1D ring. The perturbative renormalization-group argument in [3,4] that the per-

sistent current is a “universal property,” and that therefore the analysis for potential scatterers in [9] can be directly transferred to the present case, begs the question, in our opinion.

The fundamental issue at stake in our disagreement with Refs. [3,4] is whether or not spin-charge separation manifests itself in boundary effects like the persistent current. We agree with Affleck and Simon that more work is needed, including large-scale numerical simulations, to eventually settle this interesting question.

Hans-Peter Eckle
School of Physics
The University of New South Wales
Sydney, 2052, Australia

Henrik Johannesson
Institute of Theoretical Physics
Chalmers and Göteborg University
SE-412 96 Göteborg, Sweden

Charles A. Stafford
Department of Physics
University of Arizona
Tucson, Arizona 85721

Received 12 December 2001; published 13 March 2002

DOI: 10.1103/PhysRevLett.88.139702

PACS numbers: 72.10.Fk, 72.15.Qm, 73.23.Ra

- [1] I. Affleck and P. Simon, preceding Comment, Phys. Rev. Lett. **88**, 139701 (2002).
- [2] H.-P. Eckle, H. Johannesson, and C. A. Stafford, Phys. Rev. Lett. **87**, 016602 (2001).
- [3] I. Affleck and P. Simon, Phys. Rev. Lett. **86**, 2854 (2001).
- [4] P. Simon and I. Affleck, Phys. Rev. B **64**, 085308 (2001).
- [5] D. Loss, Phys. Rev. Lett. **69**, 343 (1992).
- [6] J.-L. Zhu, X. Chen, and Y. Kawazoe, Phys. Rev. B **55**, 16 300 (1997).
- [7] F. D. M. Haldane, J. Phys. C **14**, 2585 (1981).
- [8] H.-P. Eckle, H. Johannesson, and C. A. Stafford (to be published).
- [9] A. O. Gogolin and N. V. Prokof'ev, Phys. Rev. B **50**, 4921 (1994).