Calculating $dd \to \alpha\pi^0$

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Why CSB and the $dd \to \alpha\pi^0$ reaction?

Charge Symmetry = symmetry in QCD, $P_{CS} = u \leftrightarrow d$ exchange.

CS violated by quark mass difference.

Little direct evidence (mirror nuclei ^{3}He-^{3}H, $np \to np$ $A_n(\theta_n) \neq A_p(\theta_p)$ asymmetry, and $a_{nn} \neq a_{pp}$).

$dd \to \alpha\pi^0$ would give direct evidence, $P_{CS}\left|\pi^0\right\rangle = -\left|\pi^0\right\rangle$ and $\sigma \propto |A_{CSB}|^2$. No kinematic or electro-magnetic corrections needed.

Not seen despite efforts for at least 30 years, one controversial Saclay result [NPA533, 675 (1991)].

IUCF preliminary results: very promising large clean signal

Theorists: What mechanism?

Main point of this talk: Symmetries in $dd \to \alpha\pi^0$ system crucial

Spin, parity, and symmetry considerations

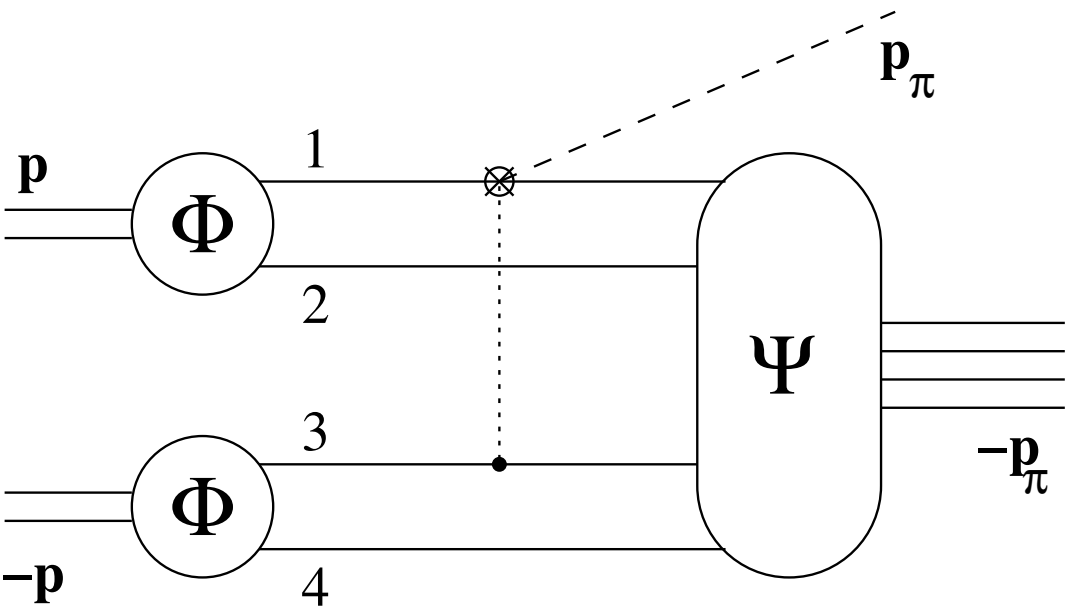
	d	d	α	π^0
Π	+	+	+	-
L	L		l	
S	0,1,2		0	
J	**L+S**		l	

identical bosons $\Rightarrow$ symmetric wf, $\Psi_{dd} = \Psi_{\text{space}}\Psi_{\text{spin}}$, cases of $(L,S) = J^\pi$ and l^π:

dd:AA	dd:SS	$\alpha\pi^0$
$(1,1) = 0^-, 1^-, 2^-$	$(0,0) = 0^+$	0^-
$(3,1) = 2^-, 3^-, 4^-$	$(0,2) = 2^+$	1^+
	$(2,0) = 2^+$	2^-
	$(2,2) = 0^+, 1^+, 2^+, 3^+, 4^+$	3^+
	$(4,0) = 4^+$	
	$(4,2) = 2^+, 3^+, 4^+$	

Allowed amplitudes: ${}^3P_0 s$, ${}^5D_1 p$, ${}^3P_2 d$, ${}^3F_2 d$, ...

Diagram and definitions



Anti-symmetrized spin-isospin wave functions

The spin-isospin parts of the wave functions are

$$\begin{aligned} |\alpha\rangle &= \frac{1}{\sqrt{2}}([(12)_1(34)_1]_0[(12)_0(34)_0]_0 \\ &\quad - [(12)_0(34)_0]_0[(12)_1(34)_1]_0) \\ |d_1\rangle &= (12)_1(12)_0 \\ |d_2\rangle &= (34)_1(34)_0 \end{aligned}$$

where the first brackets is spin and the second isospin. The total dd and $\alpha\pi^0$ states are given by

$$\begin{aligned} |DD\rangle &= \sqrt{4E_{d_1}E_{d_2}}\Phi_1\Phi_2 e^{i\mathbf{p}_d\cdot\mathbf{r}}\,|d_1\rangle\,|d_2\rangle \\ |A\Pi^0\rangle &= \sqrt{2E_\alpha}\Psi e^{i\mathbf{p}_\pi\cdot\mathbf{r}_\pi}\,|\alpha\rangle \end{aligned}$$

where $1 = \int d^3\rho|\Phi|^2 = \int d^3r d^3\rho_1 d^3\rho_2|\Psi|^2$.
Choose 2+2 coordinates r, ρ_1, ρ_2 given by

$$\begin{aligned} \mathbf{r} &= \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2 - \mathbf{r}_3 - \mathbf{r}_4) \\ \boldsymbol{\rho}_1 &= \mathbf{r}_1 - \mathbf{r}_2 \\ \boldsymbol{\rho}_2 &= \mathbf{r}_3 - \mathbf{r}_4 \end{aligned}$$

where $\sum_{i<j}(\mathbf{r}_i - \mathbf{r}_j)^2 = 4\mathbf{r}^2 + 2\boldsymbol{\rho}_1^2 + 2\boldsymbol{\rho}_2^2$.
Gaussian α wave function is given by

$$\Psi(\mathbf{r}, \boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \frac{8}{\pi^{9/4}\alpha^{9/2}} \exp\left[-\frac{1}{2\alpha^2}(4\mathbf{r}^2 + 2\boldsymbol{\rho}_1^2 + 2\boldsymbol{\rho}_2^2)\right]$$

Matrix element

The $dd \to \alpha\pi^0$ matrix element for s-wave wave functions could now be written as

$$\begin{aligned}\mathcal{M} &= \sqrt{2E_\alpha s}\int d^3r d^3\rho_1 d^3\rho_2 \Psi^\dagger(r,\rho_1,\rho_2)\Phi(\rho_1)\Phi(\rho_2) \\ &\times e^{i\mathbf{p}_d\cdot\mathbf{r}}\sum_{i<j} e^{-i\mathbf{p}_\pi\cdot\mathbf{r}_i}\left\langle\alpha\right|\bar{u}_{i'}\Gamma_i u_i V(r_{ij})\bar{u}_{j'}\Gamma_j u_j\left|dd\right\rangle\end{aligned}$$

where $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$. The integrated cross section (s-wave π) is given by

$$\sigma = \frac{4\pi}{16(2\pi)^2 s}\frac{p_\pi}{p_d}\frac{1}{9}\sum|\mathcal{M}|^2$$

One-body amplitude

An s-wave one-body operator

$$\Gamma_\pi = G_\pi \boldsymbol{\sigma} \cdot \left(\mathbf{p}_\pi - \frac{\omega}{M} \mathbf{p} \right) \vec{\tau} \cdot \vec{\pi}$$

by it self gives

$$\langle \alpha | \, \Gamma_\pi \, | dd \rangle = \sum_i G_\pi \frac{\omega}{2M} \frac{-i}{\sqrt{6}} (\mathbf{k}_i + \mathbf{k}_{i'}) \cdot (\boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_2)$$

Two-body amplitude

The pion rescattering operator

$$\Gamma_{\pi\pi} = G_{\pi\pi} \vec{\tau} \cdot \vec{\pi} \pi_3$$

together with the one-body amplitude and the Yukawa potential gives

$$\langle \alpha | \, \Gamma_{\pi\pi} V_{ij} \Gamma_\pi \, | dd \rangle = G_\pi G_{\pi\pi}$$
$$\times \quad \sum_{i<j} \langle \alpha | \, \bar{u} \vec{\tau} \cdot \vec{\pi} \pi_3 u \frac{e^{-\mu r_{ij}}}{4\pi r_{ij}} \bar{u} \boldsymbol{\sigma} \cdot (\mathbf{p}_\pi - \frac{\omega}{M} \mathbf{p}) \vec{\tau} \cdot \vec{\pi} \, | dd \rangle$$

Simplifies when homework problem is understood.

Why the homework problem vanishes

HW = calculate matrix element for the two-body amplitude containing $\boldsymbol{\sigma} \cdot \mathbf{p}_\pi$ using s-wave d and α wave functions

HW=0 because

A: The singlet-singlet isotriplet-isotriplet (d^*d^*) part of α wf doesn't contribute.
Only dd-like state of α contributes!

B: The triplet-triplet isosinglet-isosinglet (dd) part forces the pion exchange to occur inside the deuteron.
No pion exchange between the deuterons.
No momentum sharing!

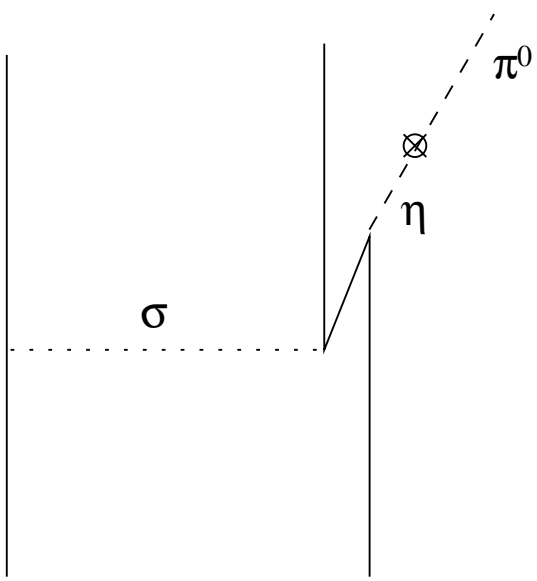
C: $\boldsymbol{\sigma} \cdot \mathbf{p}_\pi \to \boldsymbol{\sigma} \cdot i\hat{\boldsymbol{\rho}}_1 \frac{\partial}{\partial \rho_1}$, but $\int d^3\rho_1 \hat{\boldsymbol{\rho}}_1 f(\rho_1) = 0$.
$\boldsymbol{\sigma} \cdot \mathbf{p}_\pi$ does not contribute to 2-body amplitude

However, $\boldsymbol{\sigma} \cdot \mathbf{p} \to \boldsymbol{\sigma} \cdot i\hat{\mathbf{r}} \frac{\partial}{\partial \mathbf{r}}$, while $\int d^3r \hat{\mathbf{r}} e^{i\mathbf{p}_d \cdot \mathbf{r}} \Psi(r) \neq 0$
$\boldsymbol{\sigma} \cdot \mathbf{p}$ term survives, Yukawa $= e^{-\mu\rho_1}/(4\pi\rho_1)$. Small!

Best option: find momentum sharing amplitude.
Candidate: σ-meson exchange with η–π^0 mixing.

d-states in wave functions: suppressed by $\sqrt{P_d}$

σ-meson exchange amplitude



An σ-exchange z-graph with CSB η–π^0 mixing

$$g_\sigma^2 \Gamma_{\eta\pi} \frac{V_{ij}}{M} \Gamma_\pi = g_\sigma^2 G_{\eta\pi}^{CSB} \frac{e^{-m_\sigma r_{ij}}}{4\pi M r_{ij}} \frac{\omega}{2M} \boldsymbol{\sigma} \cdot (\mathbf{k}_i + \mathbf{k}_{i'})$$

where $G_{\eta\pi}^{CSB} = g_A \bar{g}_\eta \left\langle \pi^0 \right| H \left| \eta \right\rangle / m_\eta^2$ gives

$$\begin{aligned}
\mathcal{M}_\sigma &= \frac{2 g_\sigma^2 G_{\eta\pi} \omega \sqrt{2 E_\alpha s}}{4\pi M \sqrt{6} f_\pi 2M} \widehat{\mathbf{p}}_d \cdot (\boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_2) \\
&\times 4\pi \int r dr d^3\rho_1 d^3\rho_2 \left(\frac{e^{-m_\sigma \rho_1}}{\rho_1} + \frac{e^{-m_\sigma r_{13}}}{r_{13}} + \frac{e^{-m_\sigma r_{14}}}{r_{14}} \right) \\
&\times \left(\sin(p_d r) + r j_1(p_d r) \frac{\partial}{\partial r} \right) \Psi^\dagger(r, \rho_1, \rho_2) \Phi(\rho_1) \Phi(\rho_2)
\end{aligned}$$

Final expressions

One-body matrix element:

$$\mathcal{M}_\pi^{CSB} = \frac{4G_\pi\omega}{2M\sqrt{6}}\widehat{\mathbf{p}}_d\cdot(\boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_2)\sqrt{2E_\alpha s}(4\pi)^3 \times \int r dr\rho_1^2 d\rho_1\rho_2^2 d\rho_2 \sin(p_d r)\Psi^\dagger(r,\rho_1,\rho_2)\Phi(\rho_1)\Phi(\rho_2)$$

Two-body:

$$\mathcal{M}_{\pi\pi} = 4G_{\pi\pi}^{CSB}\frac{G_\pi^{CS}}{G_\pi^{CSB}}\left\langle\frac{e^{-\mu\rho_1}}{4\pi\rho_1}\right\rangle\mathcal{M}_\pi$$

σ-meson exchange:

$$\mathcal{M}_{\sigma\pi} = \frac{g_\sigma^2 G_{\eta\pi}^{CSB}}{8\pi M G_\pi^{CSB}}\left\langle\frac{e^{-m_\sigma\rho_1}}{\rho_1}+\frac{e^{-m_\sigma r_{13}}}{r_{13}}+\frac{e^{-m_\sigma r_{14}}}{r_{14}}\right\rangle\mathcal{M}_\pi$$

if

$$\sin(p_d r)\to\frac{1}{2}\left(\sin(p_d r)+r j_1(p_d r)\frac{\partial}{\partial r}\right)$$

Cross sections

The different coupling constants are defined by:

$$
\begin{aligned}
2f_\pi G_\pi^{CS} &= g_A = 1.25 \\
2f_\pi G_\pi^{CSB} &= \beta_1 = 4.7 \cdot 10^{-3} \\
2f_\pi G_{\pi\pi}^{CSB} &= \frac{1}{2f_\pi}\left(\delta m - \frac{\bar{\delta}}{2}\right) = 22 \cdot 10^{-3} \\
2f_\pi G_{\eta\pi}^{CSB} &= \beta_1 = 4.7 \cdot 10^{-3}
\end{aligned}
$$

The contributions to the cross section are

T_d	228.5 MeV	231,8 MeV
σ_π	0.43 pb	0.55 pb
$\sigma_{\pi\pi}$	0.04 pb	0.05 pb
$\sigma_{\sigma\pi}$	4.3 pb	6.1 pb

Smallness of π and $\pi\pi$ XS due to the Fourier transform $d^3r e^{i\mathbf{p}_d\cdot\mathbf{r}}\Psi(r) \propto e^{-\alpha^2 p_d^2/8} \ll 1$.

Experiment: $\sigma =?$ pb

Conclusions

- Symmetries of dd and α s-states allow only very specific spin-isospin structures of amplitudes $\Rightarrow$ simple π rescattering small

- Symmetries support coherent pion production

- Need exchange between the deuterons to share momentum,
  preliminary: σ-meson exchange gives $\sigma = 4$ pb

- Improvements and extensions:

  1. other CSB mechanisms:
     ρ–ω exchange (between d's)
     p-wave pion amplitudes (^5D_1p), tensor, Δ's?
     photon exchange

  2. realistic wave functions:
     α - Nogga
     dd - Fonseca+IUCF

  3. CSB in initial and final state wf