List of operators that contribute to $dd \to \alpha\pi^0$

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There are several scales in the problem, namely

- p/M_N, the initial momentum over the chiral symmetry breaking scale, here identified with the nucleon mass,
- m_π/M_N, where m_π denotes the pion mass,
- Δ/M_N, with Δ as the Delta–nucleon mass splitting, and
- γ, the typical nucleon momentum inside the α particle.

In addition, charge symmetry breaking can occur either via exchange of a virtual photon or via strong breaking with the corresponding scales

- $\alpha/4\pi$, the fine structure constant that appears with every exchange of a virtual photon,
- $\epsilon \simeq 1/3$ is related to the up–down mass difference.

The loop integrals—and here I also mean reducible loops—that appear with and without photon exchange are accompanied with different integrals. Thus, at this stage an independent expansion of diagrams where there is a strong charge symmetry breaking and those where is stems from the photon exchange seems unavoidable.

In oder to make three body contributions comparable to two body contributions, always the full transition matrix elements are considered. However, there are a couple of common factors anyway that should not be kept explicitly, namely, since we study a system of 4 nucleons that are bound in an α in the final state, there are always three loops that are controlled by γ. Thus, all we need to keep explicitly for an n–nucleon operator in addition to what can be read off the vertices and propagators directly is a factor $(p^3/(4\pi))^{(n-1)}$ (here we have a three dimensional integral only because we estimate the measure of a convolution integral with a wave function). This way γ does not occur explicitly in the order assignments. Please note, that this way *I do not properly account for the unitarity cut* that can occur in any pure multi nucleon state between the production operator and the wave function ... simply because I did not know how. Thus, the procedure proposed at least allows to compare production operators with different numbers of nucleons, but might assign too low an order to 3 and 4 nucleon diagrams.

We work in the following scheme [1]

$$M_N \gg p \sim \Delta \gg m_\pi \ .$$

It is this assignment that pushes the lowest order contribution from the Δ out to N^{3}LO.

1 Diagrams proportional to ϵ

1.1 Effects in the production operator

Below the $\mathbf{x}$ denotes the corresponding charge symmetry breaking operator. The order assignment refers to powers in $\chi = p/M_N \sim \sqrt{m_\pi/M_N}$, where p is the initial momentum. Charge symmetry breaking contact interactions are constructed via assuming them to scale with $\delta m \sim \epsilon m_\pi^2/M_N$, and filling in a sufficient amount of f_π and M_N to archieve the proper units as well as to force the number of f_π to match the expectations set by the LO diagram. The lowest order diagrams are shown in table 1.

1.2 Effects in the wavefunction

The next question to be addressed is starting from what order to we need to consider CSB in the wave function. To see this, the easiest is it to compare the size of the LO CSB production operator (rescattering via the Weinberg term) times the LO CSC contribution to the NN potential (e.g. one pion exchange) to the LO CSC production operator (rescattering via the Weinberg–Tomozawa term) times the LO CSB contribution to the NN scattering (I assumed this to be one pion exchange with a CSB coupling on one vertex). This analysis gives, that CSB in the wave function should be significant already in an N^{2}LO calculation!

2 Diagrams proportional to α

Although on the first glance contribution proportional to ϵ seem to be significantly larger than those proportional to α. In fact, comparing the estimated

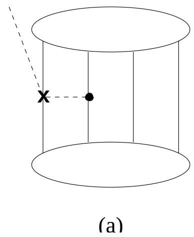
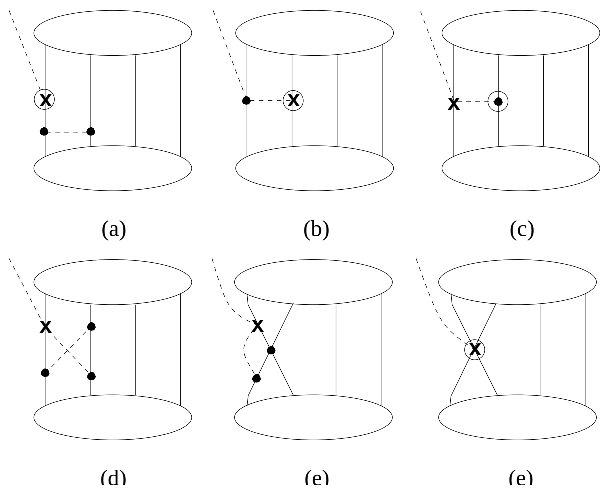
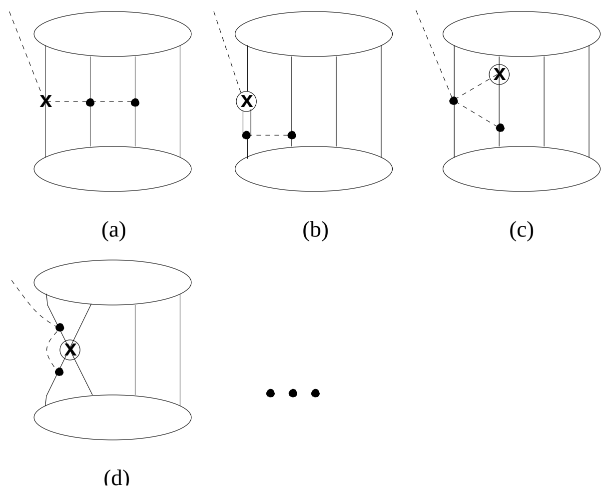
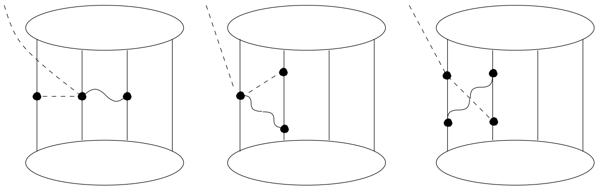
order	diagrams	comments
LO		the CSB term is the Weinberg term.
N^2LO		in diagram (a) the CSB can occur in any of the three vertices and a one pion exchange diagram can also be replaced by a 4N operator (with or without CSB)
N^3LO		this list might well be not complete; the double line in (b) denotes a Δ; in (d) also the πNN vertex could be the charge symmetry breaking one, without changing the order assigned.

Table 1
Leading order charge symmetry proprotional to ϵ.

size, dropping all factors of 2, but keeping factors of 4π, gives that the LO proportional to ϵ is enhanced by a factor of $\epsilon/(\alpha/(4\pi))(m_\pi/M_N)^{(3/2)} \sim 30$ to those proportional to α. However, the fact that the photon is massless can lead to an enhancement of the diagrams that correspond to virtual photon exchange. This needs to be seen in the application (photon loops typically produce an additional M_N/p [2]). This is why we should keep the virtual photons in explicitly.

2.1 Effects in the production operator

The leading order diagrams are given by



(a) (b) (c)

It is important to know, that in threshold kinematics (on the two body level the outgoing nucleons as well as the produced pion are at rest) the two loops cancel—in the realistic calculation we should expect some of this cancellation to survive ... diagram (a) should therefore be the one to estimate the photon effects in the production operator.

2.2 Effects in the wavefunction

The same kind of estimate as given for the contributions proportional to ϵ can be applied here as well. One finds that the electromagnetic effects in the wavefunction appear even one order below those discussed in the previous subsection. The leading piece is a simple one photon exchange between two nucleons.

References

[1] C. Hanhart and N. Kaiser, Phys. Rev. C **66** (2002) 054005

[2] G. Rupak and Xin–wei Kong, arXiv:nucl-th/0108059.