Heavy Meson Exchange CSB Operators for $dd \to {}^4\mathrm{He}\pi^0$

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Here are heavy meson exchange operators assuming $\pi - \eta$ or $\rho - \omega$ mixing for the isospin violation. The total hamiltonian is the sum of one-body, sigma, omega, and rho exchange, and rho-omega mixing contributions,

$$H_{tot} = H_1 + H_\sigma + H_\omega + H_\rho + H_{\rho-\omega}.$$

Start from the one body Hamiltonian of the homework problem,

$$H_1 = \Lambda_1 \sum_{i=1}^{4} \sigma_i \cdot p_i,$$

$$\Lambda_1 = \frac{1}{(2m_\pi)^{1/2}} f\Big(\frac{\beta_1}{g_a}\Big)\frac{1}{M}.$$

Here the conventions are from K-R with the factor $(2m_\pi)^{1/2}$ from phase space and f is the pion nucleon coupling constant $f^2 = 0.96$ and the isospin violation β_1/g_a is say -2.8×10^{-3} from $\pi-\eta$ mixing. The ith nucleon's momentum is p_i. See the 2nd homework problem.

For the sigma exchange two-body operator, replace M by $M* = M - \Sigma_{j=1}^4 g_\sigma^2 e^{-m_s r_{ij}}/4\pi r_{ij}$ and expand to first order in g_σ^2. This gives the two-body Hamiltonian H_σ,

$$H_\sigma = \Lambda_1 (\frac{g_\sigma^2}{4\pi M})\frac{1}{2}\sum_{i\neq j}\sigma_i\cdot(f_{ij}^\sigma p_i + p_i f_{ij}^\sigma),$$

$$f_{ij}^\sigma = \frac{\mathrm{e}^{-m_\sigma r_{ij}}}{r_{ij}}.$$

Note the sum over i not equal to j rather than i less than j. We use typical OBE parameters for the sigma meson $g_\sigma^2 = 110$ and $m_\sigma = 520$ MeV.

The omega exchange two-body operator, H_ω, is

$$H_\omega = -\Lambda_1 (\frac{g_\omega^2}{4\pi M})\frac{1}{2}\sum_{i\neq j}\Big\{\sigma_j\cdot(f_{ij}^\omega p_i + p_i f_{ij}^\omega) + i\sigma_i\wedge\sigma_j\cdot q_j f_{ij}^\omega\Big\},$$

$$f_{ij}^\omega = \frac{\mathrm{e}^{-m_\omega r_{ij}}}{r_{ij}}.$$

Note this has an overall minus sign and σ_j instead of σ_i compared to H_σ. Finally there is the new term involving the momentum transfered to nucleon j, q_j. We use typical OBE parameters for the omega meson $g_\omega^2 = 183$ and $m_\omega = 783$ MeV.

The rho exchange two-body operator, H_ρ, is

$$H_\rho = -\Lambda_1 (\frac{g_\rho^2}{4\pi M})\frac{1}{2}\sum_{i\neq j}\tau_i\cdot\tau_j\Big\{\sigma_j\cdot(f_{ij}^\rho p_i + p_i f_{ij}^\rho) + i(1+C_\rho)\sigma_i\wedge\sigma_j\cdot q_j f_{ij}^\rho\Big\},$$

$$f_{ij}^\rho = \frac{\mathrm{e}^{-m_\rho r_{ij}}}{r_{ij}}.$$

This has an isospin factor $\tau_i\cdot\tau_j$ which presumably makes it less important for $dd\to{}^4\mathrm{He}\pi^0$. The factor C_ρ is the (large) ratio of tensor $i\sigma_{\mu\nu}q^\nu/2M$ to vector γ_μ coupling for the rho. The rho HMEC is of order the small vector times the large tensor coupling constant and has no contributions of order the tensor coupling squared.

The rho-omega mixing two-body operator, $H_{\rho-\omega}$, is

$$H_{\rho-\omega} = -\Lambda_{\rho-\omega}(\frac{g_\rho g_\omega}{4\pi M})\frac{1}{2}\sum_{i\neq j}\Big\{(1+\tau_i^3\tau_j^3)\sigma_j\cdot(f_{ij}^{\rho\omega}p_i + p_i f_{ij}^{\rho\omega}) + i[1+\tau_i^3\tau_j^3(1+C_\rho)]\sigma_i\wedge\sigma_j\cdot q_j f_{ij}^{\rho\omega}\Big\},$$

$$\Lambda_{\rho-\omega} = \frac{1}{(2m_\pi)^{1/2}} f\Big(\frac{<\rho|H\omega>}{m_\omega^2}\Big)\frac{1}{M},$$

$$f_{ij}^{\rho\omega} = \frac{m_\omega^2}{m_\omega^2 - m_\rho^2}(\frac{\mathrm{e}^{-m_\rho r_{ij}}}{r_{ij}} - \frac{\mathrm{e}^{-m_\omega r_{ij}}}{r_{ij}}).$$

The isospin independent part of this $\rho-\omega$ operator is only of order the small rho vector coupling. *Therefore $\rho-\omega$ mixing is expected to be somewhat smaller for $dd \to {}^4\mathrm{He}\pi^0$ then σ or ω heavy meson exchange currents (however it is the same order of magnitiude because $\pi-\eta$ mixing involves the small ratio of eta to pi-nucleon couplings).* Note there is a $\tau_i^3\tau_j^3$ term that does involve the large tensor rho coupling. This could give a large contribution to CSB for $np \to d\pi^0$.