Computing $\sigma(DD \to \alpha\pi^0)$ and Charge Symmetry Breaking-HW problems

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Our report on the workshop states that,

"A homework problem was developed. This involves using the simplest possible DD and α wave functions to provide a semi-analytic calculation to test the use of more complicated methods."

The following is meant to describe the homework problem in more detail. It turned out that the first version of the problem led to a null result for the threshold cross section, so it was necessary to define new versions.

The problem is defined by stating the initial and final wave functions and the interaction operator. For threshold production of a pion the initial state must be $J^\pi = 0^-$. We write the initial wave function as

$$\begin{aligned}
|I\rangle &= |\mathbf{k}, M_1, M_2\rangle \\
&= \frac{1}{\sqrt{6}}(1 + P_{23} + P_{24})\left(e^{i\mathbf{k}\cdot(\mathbf{R_1}-\mathbf{R_2})} + (-1)^S e^{-i\mathbf{k}\cdot(\mathbf{R_1}-\mathbf{R_2})}\right)\sum_{S,M_s}|S,M_s\rangle\langle S,M_s|1M_1,1M_2\rangle, \quad (1)\\
&\quad |1M_1,1M_2\rangle \equiv |D_{1,2}(M_1)\otimes D_{3,4}(M_2)\rangle, \quad (2)
\end{aligned}$$

where $S = 1$ for the 0^- state, the notation $D_{1,2}(M_1)$ stands for a deuteron, made of nucleons 1 and 2, in a total spin state of M_1, and P_{23} is a particle interchange operator. For the homework problem we take the deuteron to be in an S-state only.

The pion production operator is taken (at threshold, and neglecting terms of order m_π/m_N) for the homework problem as due to the quark mass difference effects [1]

$$\mathcal{O} \equiv \sum_{i<j} \mathcal{O}_{i,j}, \quad (3)$$

$$\begin{aligned}
\mathcal{O}_{i,j} = &-\frac{f}{4\pi\mu F_\pi^2\sqrt{2\mu}}[\delta m_N(\boldsymbol{\tau}^{(i)}\cdot\boldsymbol{\tau}^{(j)} + \tau_3^{(i)}\tau_3^{(j)}) - \bar{\delta} m_N(\boldsymbol{\tau}^{(i)}\cdot\boldsymbol{\tau}^{(j)} - \tau_3^{(i)}\tau_3^{(j)})] \\
&(\boldsymbol{\sigma}^{(i)} - \boldsymbol{\sigma}^{(j)})\cdot(-i\hat{\boldsymbol{r}})\frac{d}{dr}\left(\frac{e^{-\mu r}}{r}\right), \quad (4)
\end{aligned}$$

$$\mathbf{r} \equiv \mathbf{r}_i - \mathbf{r}_j \quad (5)$$

where $F_\pi = 186$ MeV is the pion decay constant, μ is the pion mass, and f is the pion-nucleon coupling constant ($f = g_A\mu/F_\pi$ in leading order; $f^2/4\pi = 0.075$ is used here). The parameters, δm_N and $\bar{\delta} m_N$, account for the effect of the current quark mass difference $m_d - m_u$ and the electromagnetic contribution. The sum of these is fixed by the nucleon mass difference $m_n - m_p = 1.3$ MeV. We know $\delta m_N \propto m_d - m_u$ and $m_\pi^2 \propto m_d + m_u$, so $\delta m_N = O(\varepsilon m_\pi^2/M_{QCD}) \sim 7$ MeV. The electromagnetic term $\bar{\delta} m_N$, long known to be negative [2], should be proportional to the fine structure constant α, so that $\bar{\delta} m_N = -O(\alpha M_{QCD}/\pi) \sim -2$ MeV. Estimates such as these cannot be trusted to better than a factor of a few, and more precise estimates can only be made using models. Most models estimate that $\bar{\delta} m_N \approx -\alpha/R_N \simeq -1.5$ MeV [3], in which case $\delta m_N \simeq 3$ MeV, which is also consistent with the results of Gasser and Leutwyler [4]. Please see [1] for more details.

The final state wave function is given by

$$|F\rangle = |\alpha\rangle \otimes |\pi^0\rangle \tag{6}$$

$$\langle \mathbf{r}_i, s_i, t_i|\alpha\rangle = Ne^{-b\left(\xi_1^2+\xi_2^2+\xi_3^2\right)}|S,T\rangle \tag{7}$$

$$|S,T\rangle = \frac{1}{\sqrt{2}}\left(|\phi'\rangle|\chi''\rangle - |\phi''\rangle|\chi'\rangle\right), \tag{8}$$

where $|\phi'(\chi')\rangle$ is the spin (isospin) wave function for particles (1,2) coupled to zero and (3,4) coupled to zero; and, $|\phi''(\chi'')\rangle$ is the spin (isospin) wave function for particles (1,2) coupled to unity and (3,4) coupled to unity:

$$|\phi'\rangle = |S=0\rangle_{1,2} \otimes |S=0\rangle_{3,4} \tag{9}$$

$$|\phi''\rangle = \frac{1}{\sqrt{3}}\left[|11\rangle_{1,2} \otimes |1-1\rangle_{3,4} - |1-1\rangle_{1,2} \otimes |11\rangle_{3,4} - |10\rangle_{1,2} \otimes |10\rangle_{3,4}\right]. \tag{10}$$

The spatial variables are given by

$$\xi_1 = \frac{\mathbf{r}_1 - \mathbf{r}_2}{\sqrt{2}} \tag{11}$$

$$\xi_2 = \frac{\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3}{\sqrt{6}} \tag{12}$$

$$\xi_3 = \frac{\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 - 3\mathbf{r}_4}{\sqrt{12}} \tag{13}$$

$$\xi_4 = \frac{\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4}{4}. \tag{14}$$

The initial and final wave functions and the interaction operator for the homework problem have been specified. The goal is to calculate the threshold cross section. We are now in a position to state several problems of increasing complexity.

- Show that the cross section for the defined problem, with only S-states in the deuteron and ^{4}He vanishes.
- Show that including the effects of tensor force either in the deuteron, ^{4}He, or initial state interactions leads to a non-zero result.
- Define a new version of the pion production operator, $\mathcal{O}^{SM}$ with

$$\mathcal{O}^{SM}_{i,j} \equiv \mathcal{O}_{i,j}\ \epsilon S_{12}(i,j) \tag{15}$$

$$S_{12}(i,j) \equiv 3\boldsymbol{\sigma}_i \cdot \widehat{\mathbf{r}}_i\ \boldsymbol{\sigma}_j \cdot \widehat{\mathbf{r}}_j - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, \tag{16}$$

where ϵ is a dimensionless constant, and compute the related cross section.

- Show that the interaction of (15) approximately simulates the effects of the initial state tensor interaction between one nucleon in one deuteron and another in the second deuteron. Estimate the value ϵ.
- An alternate problem is to compute the cross section using the interaction (5), but including the effects of the deuteron D-state.

We hope that putting this homework problem on the bulletin board will help to keep the enthusiasm which everyone showed at the meeting.

REFERENCES

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