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Second Homework Problem. One-body Pion Production.

Calculate the $d + d \to {}^4He\,\pi^0$ cross section for a simple one-body π production operator and very simple wave functions.

Purpose: (1) Make a simple NONSZERO calculation to insure we all agree on phase space factors and conventions.

(2) Provide a simple benchmark to compare larger two-body calculations.

(3) Allow Nogga to check how the one-body contribution depends on high momentum components in the ${}^4He$ wave function.

A) One-body operator for s-wave pions

Conventions from Koltun + Reitan, Phys Rev 141 (1996) 1413.

$$H_{1-body} = \frac{1}{\sqrt{2m_\pi}} f\left(\frac{\beta_1}{g_a}\right) \sum_{i=1}^{4} \sigma_i \left[\frac{p_i + p_i'}{2m}\right]$$

Here $\frac{\beta_1}{ga}$ is the iso spin violating coupling of a pion to the nucleon. The factor $(2m_\pi)^{-1/2}$ is from the π phase space. Note H_1 is of recoil order m_π/m compared to a coupling for p-wave pions. Finally p_i is inital and p_i' is final nucleon momentum.

B) For $\pi - \eta$ mixing

$$\frac{\beta_1}{g_a} = \frac{g_\eta}{g_\pi} \frac{< \pi H \eta >}{m_\eta^2}$$

$$= \frac{-3.5 \times, 10^{-3}}{g_a} = -2.8\,x\,10^{-3}$$

$< \pi\, H\, \eta >= \, -4200\ MeV^2$, $g_a = 1.26$, see van Kolck et al, nucl-th/9601009

C) Simple Gaussian Wave Functions

$$\Psi_{H_e} \; = \; \left[\frac{64}{\pi^{9/2}\alpha^9}\right]^{1/2} \; e^{-\left(2r^2+\rho_1^2+\rho_2^2\right)/\alpha^2}$$

$$\alpha = 2.77\, Fm \qquad \begin{array}{c} r = (r_1 + r_2 - r_3 - r_4)/2 \\ \rho_1 = r_1 - r_2 \\ \rho_3 = r_3 - r_4 \end{array}$$

$$\Phi_D(\rho) = \left[\frac{1}{\pi^{3/2}\beta^3}\right]^{1/2} \; e^{-\rho^2/2\beta^2} \quad \beta = 3.21\ Fm$$

D) <u>Answer</u>

$$\sigma \; = \; \frac{16\pi^{1/2}}{27}\, f^2 \left(\frac{\beta_1}{g_a}\right)^2 \left(\frac{k_\pi k}{m}\right) \left(\frac{1}{\alpha^2} + \frac{1}{2\beta^2}\right)^{-6} \frac{e}{\alpha^3\beta^6}^{-\frac{k^2\alpha^2}{4}}$$

k = relative momentum of two deuterons in cm system $\approx$460 MeV

$k_\pi \; \approx \pi$ momentum = 19 MeV

$f^2 = 0.96$

$\left(\frac{\beta_1}{g_a}\right) \; = \; -2.8\times, 10^{-3}$

$\sigma = 0.15\ pb$