Phase space for the $\sigma(DD \to \alpha\pi^0)$ Reaction

Gerald A. Miller

The problem is defined by stating the initial and final wave functions and the interaction operator. For threshold production of a pion the initial state must be $J^\pi = 0^-$. We write the initial wave function as

$$
\begin{aligned}
|I\rangle &= |\mathbf{k}, M_1, M_2\rangle \\
&= \frac{1}{\sqrt{6}}(1 + P_{23} + P_{24})\left(e^{i\mathbf{k}\cdot(\mathbf{R_1}-\mathbf{R_2})} + (-1)^S e^{-i\mathbf{k}\cdot(\mathbf{R_1}-\mathbf{R_2})}\right)\sum_{S,M_s}|S, M_s\rangle\langle S, M_s|1M_1, 1M_2\rangle, \quad (1) \\
&\quad |1M_1, 1M_2\rangle \equiv |D_{1,2}(M_1) \otimes D_{3,4}(M_2)\rangle, \quad (2)
\end{aligned}
$$

where $S = 1$ for the 0^- state, the notation $D_{1,2}(M_1)$ stands for a deuteron, made of nucleons 1 and 2, in a total spin state of M_1, and P_{23} is a particle interchange operator. For the homework problem we take the deuteron to be in an S-state only.

The pion production operator is taken (at threshold, and neglecting terms of order m_π/m_N) for the homework problem as due to the quark mass difference effects [1]

$$
\begin{aligned}
\mathcal{O} &\equiv \sum_{i<j} \mathcal{O}_{i,j}, \quad (3) \\
\mathcal{O}_{i,j} &= -\frac{f}{4\pi\mu F_\pi^2\sqrt{2\mu}}[\delta m_N(\boldsymbol{\tau}^{(i)}\cdot\boldsymbol{\tau}^{(j)} + \tau_3^{(i)}\tau_3^{(j)}) - \bar{\delta} m_N(\boldsymbol{\tau}^{(i)}\cdot\boldsymbol{\tau}^{(j)} - \tau_3^{(i)}\tau_3^{(j)})] \\
&\quad (\boldsymbol{\sigma}^{(i)} - \boldsymbol{\sigma}^{(j)})\cdot(-i\hat{\boldsymbol{r}})\frac{d}{dr}\left(\frac{e^{-\mu r}}{r}\right), \quad (4) \\
\mathbf{r} &\equiv \mathbf{r}_i - \mathbf{r}_j \quad (5)
\end{aligned}
$$

where $F_\pi = 186$ MeV is the pion decay constant, μ is the pion mass, and f is the pion-nucleon coupling constant ($f = g_A\mu/F_\pi$ in leading order; $f^2/4\pi = 0.075$ is used here). The parameters, δm_N and $\bar{\delta} m_N$, account for the effect of the current quark mass difference $m_d - m_u$ and the electromagnetic contribution. The sum of these is fixed by the nucleon mass difference $m_n - m_p = 1.3$ MeV. We know $\delta m_N \propto m_d - m_u$ and $m_\pi^2 \propto m_d + m_u$, so $\delta m_N = O(\varepsilon m_\pi^2/M_{QCD}) \sim 7$ MeV. The electromagnetic term $\bar{\delta} m_N$, long known to be negative [2], should be proportional to the fine structure constant α, so that $\bar{\delta} m_N = -O(\alpha M_{QCD}/\pi) \sim -2$ MeV.

Note the appearance of the factor $\frac{1}{\sqrt{2\mu}}$. This is a phase space factor, valid only at threshold. Its appearance has a historical origin, rooted in models which treat the nucleon as static. It is better to leave this out of the interaction, and to use the relativistic phase space. Thus define

$$
V \equiv \mathcal{O}\sqrt{2\mu}. \quad (6)
$$

Such replacements should be made in all operators.

The final state wave function is given by

$$|F\rangle = |\alpha\rangle \otimes |\pi^0\rangle \tag{7}$$

$$\langle \mathbf{r}_i, s_i, t_i|\alpha\rangle = Ne^{-b\left(\xi_1^2+\xi_2^2+\xi_3^2\right)}|S,T\rangle \tag{8}$$

$$|S,T\rangle = \frac{1}{\sqrt{2}}\left(|\phi'\rangle|\chi''\rangle - |\phi''\rangle|\chi'\rangle\right), \tag{9}$$

where $|\phi'(\chi')\rangle$ is the spin (isospin) wave function for particles (1,2) coupled to zero and (3,4) coupled to zero; and, $|\phi''(\chi'')\rangle$ is the spin (isospin) wave function for particles (1,2) coupled to unity and (3,4) coupled to unity:

$$|\phi'\rangle = |S=0\rangle_{1,2} \otimes |S=0\rangle_{3,4} \tag{10}$$

$$|\phi''\rangle = \frac{1}{\sqrt{3}}\left[|11\rangle_{1,2} \otimes |1-1\rangle_{3,4} - |1-1\rangle_{1,2} \otimes |11\rangle_{3,4} - |10\rangle_{1,2} \otimes |10\rangle_{3,4}\right]. \tag{11}$$

The value of b is chosen to yield the experimental vale of the rms radius, which is 1.67 fm.

The spatial variables are given by

$$\xi_1 = \frac{\mathbf{r}_1 - \mathbf{r}_2}{\sqrt{2}} \tag{12}$$

$$\xi_2 = \frac{\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3}{\sqrt{6}} \tag{13}$$

$$\xi_3 = \frac{\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 - 3\mathbf{r}_4}{\sqrt{12}} \tag{14}$$

$$\xi_4 = \frac{\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4}{4}. \tag{15}$$

The initial and final wave functions and the interaction operator for the homework problem have been specified. The goal is to calculate the threshold cross section.

Now finally, we come to the phase space factors and cross section

$$\mathcal{M} \equiv \langle F|V|I\rangle \tag{16}$$

$$d\sigma = (2\pi)^4|\mathcal{M}|^2\delta^{(4)}(p_1 + p_2 - p_\pi - p_\alpha)\frac{d^3p_\alpha d^3p_\pi 2M_\alpha}{(2\pi)^6\ 2E_\alpha 2E_\pi}\frac{4M_d^2}{4\sqrt{(p_1\cdot p_2)^2 - M_d^2}}, \tag{17}$$

$$\frac{d\sigma}{d\Omega_\pi} = \frac{|\mathcal{M}|^2}{(2\pi)^2}\frac{M_\alpha}{E_\alpha}\frac{p_\pi}{2E_\pi}\frac{M_d^2}{\sqrt{T_L^2M_d^2 + 2T_LM_D^3}} \tag{18}$$

where $p_{1,2}$ are the momenta of the two deuterons in the initial state and T_L is the deuteron kinetic energy in the laboratory frame. This is the angular distribution in the cm frame.

REFERENCES

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